Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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8874233119

MATHEMATICS **9709/62**

Paper 6 Probability & Statistics 2 **October/November 2021**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

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1 The mass, in kilograms, of a block of cheese sold in a supermarket is denoted by the random variable M. The masses of a random sample of 40 blocks are summarised as follows.

$$n = 40 \qquad \Sigma m = 20.50 \qquad \Sigma m^2 = 10.7280$$

(a) Calculate unbiased estimates of the population mean and variance of M. [3]

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(b) The price, \$$P$, of a block of cheese of mass M kg is found using the formula $P = 11M + 0.50$.

Find estimates of the population mean and variance of P. [3]

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2 Andy and Jessica are doing a survey about musical preferences. They plan to choose a representative sample of six students from the 256 students at their college.

(a) Andy suggests that they go to the music building during the lunch hour and choose six students at random from the students who are there.

Give a reason why this method is unsatisfactory. [1]

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(b) Jessica decides to use another method. She numbers all the students in the college from 1 to 256. Then she uses her calculator and generates the following random numbers.

204393 162007 204028 587119 207395

From these numbers, she obtains six student numbers. The first three of her student numbers are 204, 162 and 7.

Continue Jessica's method to obtain the next three student numbers. [2]

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3 The probability that a certain spinner lands on red on any spin is p. The spinner is spun 140 times and it lands on red 35 times.

(a) Find an approximate 96% confidence interval for p. [3]

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From three further experiments, Jack finds a 90% confidence interval, a 95% confidence interval and a 99% confidence interval for p.

(b) Find the probability that exactly two of these confidence intervals contain the true value of p. [3]

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4 A certain kind of firework is supposed to last for 30 seconds, on average, after it is lit. An inspector suspects that the fireworks actually last a shorter time than this, on average. He takes a random sample of 100 fireworks of this kind. Each firework in the sample is lit and the time it lasts is noted.

(a) Give a reason why it is necessary to take a sample rather than testing all the fireworks of this kind. [1]

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It is given that the population standard deviation of the times that fireworks of this kind last is 5 seconds.

(b) The mean time lasted by the 100 fireworks in the sample is found to be 29 seconds.

Test the inspector's suspicion at the 1% significance level. [5]

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(c) State with a reason whether the Central Limit theorem was needed in the solution to part **(b)**. [1]

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5 In a certain large document, typing errors occur at random and at a constant mean rate of 0.2 per page.

(a) Find the probability that there are fewer than 3 typing errors in 10 randomly chosen pages. [2]

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(b) Use an approximating distribution to find the probability that there are more than 50 typing errors in 200 randomly chosen pages. [4]

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In the same document, formatting errors occur at random and at a constant mean rate of 0.3 per page.

(c) Find the probability that the total number of typing and formatting errors in 20 randomly chosen pages is between 8 and 11 inclusive. [3]

6 A machine is supposed to produce random digits. Bob thinks that the machine is not fair and that the probability of it producing the digit 0 is less than $\frac{1}{10}$. In order to test his suspicion he notes the number of times the digit 0 occurs in 30 digits produced by the machine. He carries out a test at the 10% significance level.

(a) State suitable null and alternative hypotheses. [1]

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(b) Find the rejection region for the test. [4]

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(c) State the probability of a Type I error. [1]

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It is now given that the machine actually produces a 0 once in every 40 digits, on average.

(d) Find the probability of a Type II error. [3]

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(e) Explain the meaning of a Type II error in this context. [1]

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7 **(a)** The probability density function of the random variable X is given by

$$f(x) = \begin{cases} kx(4-x) & 0 \leqslant x \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(i) Show that $k = \frac{3}{16}$. [3]

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(ii) Find $\mathrm{E}(X)$. [3]

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(b) The random variable Y has the following properties.

- Y takes values between 0 and 5 only.
- The probability density function of Y is symmetrical.

Given that $P(Y < a) = 0.2$, find $P(2.5 < Y < 5 - a)$ illustrating your method with a sketch on the axes provided. [3]



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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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