Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

7688702067

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

© UCLES 2020

BLANK PAGE

1 Solve the equation

$$\ln(1 + e^{-3x}) = 2.$$

Give the answer correct to 3 decimal places. [3]

2 **(a)** Expand $\sqrt[3]{1+6x}$ in ascending powers of x, up to and including the term in x^3, simplifying the coefficients. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) State the set of values of x for which the expansion is valid. [1]

.....

.....

.....

.....

.....

© UCLES 2020

3 The variables x and y satisfy the relation $2^y = 3^{1-2x}$.

(a) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the exact x-coordinate of the point of intersection of this line with the line $y = 3x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

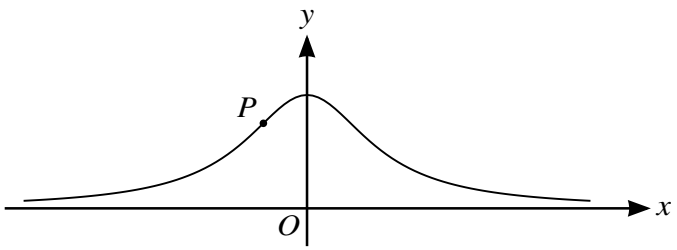
.....

4 **(a)** Show that the equation $\tan(\theta + 60°) = 2\cot\theta$ can be written in the form

$$\tan^2\theta + 3\sqrt{3}\tan\theta - 2 = 0.$$ [3]

(b) Hence solve the equation $\tan(\theta + 60°) = 2\cot\theta$, for $0° < \theta < 180°$. [3]

5



The diagram shows the curve with parametric equations

$$x = \tan\theta, \quad y = \cos^2\theta,$$

for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

(a) Show that the gradient of the curve at the point with parameter θ is $-2\sin\theta\cos^3\theta$. [3]

The gradient of the curve has its maximum value at the point P.

(b) Find the exact value of the x-coordinate of P. [4]

6 The complex number u is defined by

$$u = \frac{7 + \mathrm{i}}{1 - \mathrm{i}}.$$

(a) Express u in the form $x + \mathrm{i}y$, where x and y are real. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show on a sketch of an Argand diagram the points A, B and C representing u, $7 + \mathrm{i}$ and $1 - \mathrm{i}$ respectively. [2]

(c) By considering the arguments of $7 + i$ and $1 - i$, show that

$$\tan^{-1}\left(\tfrac{4}{3}\right) = \tan^{-1}\left(\tfrac{1}{7}\right) + \tfrac{1}{4}\pi. \qquad [3]$$

7 The variables x and t satisfy the differential equation

$$e^{3t}\frac{dx}{dt} = \cos^2 2x,$$

for $t \geqslant 0$. It is given that $x = 0$ when $t = 0$.

(a) Solve the differential equation and obtain an expression for x in terms of t. [7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) State what happens to the value of x when t tends to infinity. [1]

.....

.....

.....

.....

.....

.....

8 With respect to the origin O, the position vectors of the points A, B, C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

(a) Show that $AB = 2CD$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the angle between the directions of $\overrightarrow{AB}$ and $\overrightarrow{CD}$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

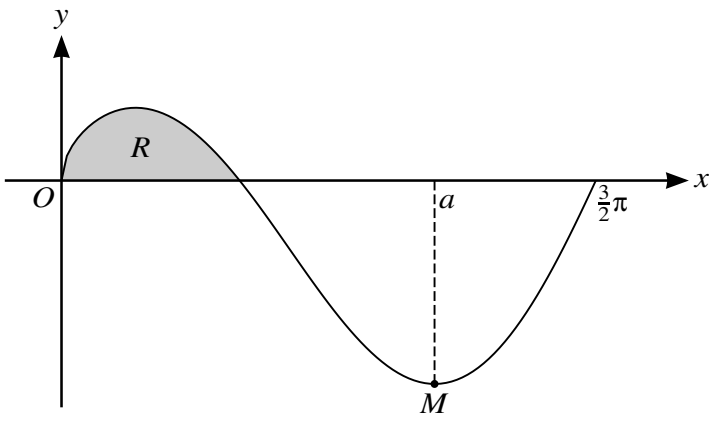
(c) Show that the line through A and B does not intersect the line through C and D. [4]

9 Let $\mathrm{f}(x) = \dfrac{7x + 18}{(3x + 2)(x^2 + 4)}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence find the exact value of $\int_0^2 \mathrm{f}(x)\,\mathrm{d}x$. [6]

10



The diagram shows the curve $y = \sqrt{x}\cos x$, for $0 \leqslant x \leqslant \frac{3}{2}\pi$, and its minimum point M, where $x = a$. The shaded region between the curve and the x-axis is denoted by R.

(a) Show that a satisfies the equation $\tan a = \dfrac{1}{2a}$. [3]

.....

.....

.....

.....

.....

.....

(b) The sequence of values given by the iterative formula $a_{n+1} = \pi + \tan^{-1}\left(\dfrac{1}{2a_n}\right)$, with initial value $x_1 = 3$, converges to a.

Use this formula to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

© UCLES 2020

(c) Find the volume of the solid obtained when the region R is rotated completely about the x-axis. Give your answer in terms of π. [6]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.