Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

8697469738

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **October/November 2020**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

© UCLES 2020

1 **(a)** Express $x^2 + 6x + 5$ in the form $(x + a)^2 + b$, where a and b are constants. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) The curve with equation $y = x^2$ is transformed to the curve with equation $y = x^2 + 6x + 5$.

Describe fully the transformation(s) involved. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2020

2 The function f is defined by $\mathrm{f}(x) = \dfrac{2}{(x+2)^2}$ for $x > -2$.

(a) Find $\displaystyle\int_1^\infty \mathrm{f}(x)\,\mathrm{d}x$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) The equation of a curve is such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x)$. It is given that the point $(-1, -1)$ lies on the curve.

Find the equation of the curve. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

3 Solve the equation $3\tan^2\theta + 1 = \dfrac{2}{\tan^2\theta}$ for $0° < \theta < 180°$. [5]

4 A curve has equation $y = 3x^2 - 4x + 4$ and a straight line has equation $y = mx + m - 1$, where m is a constant.

Find the set of values of m for which the curve and the line have two distinct points of intersection. [5]

5 In the expansion of $(a + bx)^7$, where a and b are non-zero constants, the coefficients of x, x^2 and x^4 are the first, second and third terms respectively of a geometric progression.

Find the value of $\frac{a}{b}$. [5]

6 The function f is defined by $f(x) = \frac{2x}{3x-1}$ for $x > \frac{1}{3}$.

(a) Find an expression for $f^{-1}(x)$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that $\frac{2}{3} + \frac{2}{3(3x-1)}$ can be expressed as $\frac{2x}{3x-1}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

(c) State the range of f. [1]

.....

.....

.....

7 The first and second terms of an arithmetic progression are $\frac{1}{\cos^2\theta}$ and $-\frac{\tan^2\theta}{\cos^2\theta}$, respectively, where $0 < \theta < \frac{1}{2}\pi$.

(a) Show that the common difference is $-\frac{1}{\cos^4\theta}$. [4]

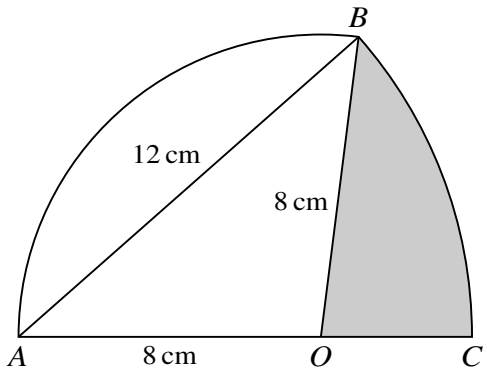
(b) Find the exact value of the 13th term when $\theta = \frac{1}{6}\pi$. [3]

8 The equation of a curve is $y = 2x + 1 + \dfrac{1}{2x+1}$ for $x > -\frac{1}{2}$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$. [3]

(b) Find the coordinates of the stationary point and determine the nature of the stationary point. [5]

9



In the diagram, arc AB is part of a circle with centre O and radius 8 cm. Arc BC is part of a circle with centre A and radius 12 cm, where AOC is a straight line.

(a) Find angle BAO in radians. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the area of the shaded region. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the perimeter of the shaded region. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

10 A curve has equation $y = \frac{1}{k}x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2}$ where $x > 0$ and k is a positive constant.

(a) It is given that when $x = \frac{1}{4}$, the gradient of the curve is 3.

Find the value of k. [4]

(b) It is given instead that $\int_{\frac{1}{4}k^2}^{k^2} \left(\frac{1}{k} x^{\frac{1}{2}} + x^{-\frac{1}{2}} + \frac{1}{k^2} \right) \mathrm{d}x = \frac{13}{12}$.

Find the value of k. [5]

11 A circle with centre C has equation $(x - 8)^2 + (y - 4)^2 = 100$.

(a) Show that the point $T\,(-6, 6)$ is outside the circle. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Two tangents from T to the circle are drawn.

(b) Show that the angle between one of the tangents and CT is exactly 45°. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The two tangents touch the circle at A and B.

(c) Find the equation of the line AB, giving your answer in the form $y = mx + c$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) Find the x-coordinates of A and B. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2020

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2020

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.