Cambridge International AS & A Level

**Cambridge Assessment International Education**
Cambridge International Advanced Subsidiary and Advanced Level

**MATHEMATICS** **9709/63**

Paper 6 **October/November 2019**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2019 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.



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## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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## Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

### Types of mark

**M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

**A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

**B** Mark for a correct result or statement independent of method marks.

**DM** or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

**FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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**Abbreviations**

| | |
|---|---|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent |
| AG | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid) |
| CAO | Correct Answer Only (emphasising that no “follow through” from a previous error is allowed) |
| CWO | Correct Working Only |
| ISW | Ignore Subsequent Working |
| SOI | Seen Or Implied |
| SC | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW | Without Wrong Working |
| AWRT | Answer Which Rounds To |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(i) | $\frac{120}{300} = 0.4$ | **B1** | OE |
| | | **1** | |
| 1(ii) | P(male) × P(not piano) = $\frac{160}{300} \times \frac{225}{300}$ $\left(\frac{8}{15} \times \frac{3}{4}\right) = \frac{2}{5}$ | **M1** | P(M) × P(P') seen<br>Can be unsimplified but the events must be named in a product |
| | As P(male ∩ not piano) also = $\frac{120}{300} = \frac{2}{5}$<br><br>The events are **Independent** | **A1** | Numerical comparison and correct conclusion |
| | **Alternative method for question 1(ii)** | | |
| | P(male ∩ not piano) = $\frac{120}{300}$; P(not piano) = $\frac{225}{300}$ | **M1** | P(M\|P') or P(P'\|M) unsimplified seen with *their* probs with correctly named events |
| | P(M \| not piano) = $\frac{\frac{120}{300}}{\frac{225}{300}} = \frac{120}{225} = \frac{8}{15}$ = P(male)<br>**or**<br>P(not piano \| M) = $\frac{\frac{120}{300}}{\frac{160}{300}} = \frac{120}{160} = \frac{3}{4}$ = P(not piano)<br><br>Therefore the events are **Independent** | **A1** | Numerical comparison with P(M) or P(P') and correct conclusion |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(i) | $\frac{9!}{2!3!} = 30240$ | **B1** | 9! Divided by at least one of 2! or 3! |
| | | **B1** | Exact value |
| | | **2** | |
| 2(ii) | D _ _ _ _ _ _ _ R: $\frac{7!}{2!2!} = 1260$<br>D _ _ _ _ _ _ _ O: $\frac{7!}{3!} = 840$ | **B1** | 7! Seen alone or as numerator in a term, can be multiplied not + or – |
| | | **B1** | One term correct, unsimplified |
| | Total = 2100 | **B1** | Final answer |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(i) | $3A\ 2D\ 2M$ : ${}^6C_3 \times {}^5C_2 \times {}^4C_2$ (= 1200)<br>$4A\ 2D\ 1M$ : ${}^6C_4 \times {}^5C_2 \times {}^4C_1$ (= 600)<br>$3A\ 3D\ 1M$ : ${}^6C_3 \times {}^5C_3 \times {}^4C_1$ (= 800) | **M1** | ${}^6C_x \times {}^5C_y \times {}^4C_z$, $x + y + z = 7$ |
| | | **A1** | 2 correct products, allow unsimplified |
| | | **M1** | Summing their totals for 3 correct scenarios only |
| | Total = 2600 | **A1** | Correct answer<br>**SC1** ${}^6C_3 \times {}^5C_2 \times {}^4C_1 \times {}^9C_1 = 7200$ |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(ii) | ${}^7C_4 \times 1$ | **B1** | ${}^7C_3$ or ${}^7C_4$ seen anywhere |
| | 35 | **B1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(i) | $P(h < 148) = 0.67$ | **B1** | $z = \pm 0.44$ seen |
| | $\frac{h-148}{8} = 0.44$ | **M1** | $z$-value $= \pm \frac{(h-148)}{8}$ |
| | $151.52 \approx 152$ | **A1** | CAO |
| | | **3** | |
| 4(ii) | $P(144 < X < 152) = P\left(\frac{144-148}{8} < Z < \frac{152-148}{8}\right)$ | **M1** | Using ± standardisation formula for either 144 or 152, $\mu = 148$, $\sigma = 8$ and no continuity correction, allow $\sigma^2$ or $\sqrt{\sigma}$ |
| | $= P\left(-\frac{1}{2} < Z < \frac{1}{2}\right) = 0.6915 - (1 - 0.6915) = 2 \times 0.6915 - 1$ | **M1** | Correct final area legitimately obtained from phi(*their* $z_2$) – phi(*their* $z_1$) |
| | $= 0.383$ | **A1** | Final probability answer |
| | $0.383 \times 120 = 45.96$<br>Accept 45 or 46 only | **B1FT** | *Their* prob (to 3 or 4 sf) × 120, rounded to a whole number or truncated |
| | | **4** | |

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| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(i) | Correct labels and scales | **B1** | Axes labelled 'cumulative frequency' (or cf) and 'time (or t) [in] min(utes)', linear scales from 0 to 90 and 0 to 200 with at least 3 values marked on each axis. |
| | 7 correctly plotted points above upper boundaries joined in a curve or line segments | **B1** | (0, 0); (10, 16); (20, 50); (30, 106); (50, 146); (70,176); (90,200) |
| | | **2** | |
| 5(ii) | 29 | **B1** | 28 ⩽ median ⩽ 30 |
| | | **1** | |
| 5(iii) | 120 seen | **M1** | For seeing 120 in a calculation or marked on the graph |
| | 37 | **A1FT** | 36 ⩽ Ans ⩽ 39 or FT from *their* graph<br>**SC1** unsupported answer in range |
| | | **2** | |
| 5(iv) | Frequencies 16 34 56 40 30 24 | **B1** | Seen. Allow unsimplified |
| | Est. Mean = $\frac{5\times16+15\times34+25\times56+40\times40+60\times30+80\times24}{200}$ | **M1** | At least 4 correct midpoints (5, 15, 25, 40, 60, 80) used in a calculation |
| | $\frac{7310}{200}$ | **M1** | Summing products of *their* 6 mid-points (not lower or upper bound or class width) × *their* frequencies / 200 (or *their* ∑f), unsimplified |
| | 36.55 | **A1** | Accept 36.6 |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(i) | $P(RR) = \frac{3}{8} \times \frac{2}{7} = \frac{3}{28}$ | **B1** | OE |
| | | **1** | |
| 6(ii) | P(RW) + P(WR)<br>$\frac{3}{8} \times \frac{5}{7} + \frac{5}{8} \times \frac{3}{7}$ | **M1** | Method shown, numerical calculations identified, may include replacements |
| | $= \frac{15}{28}$ | **A1** | AG, Fully correct calculations |
| | **Alternative method for question 6(ii)** | | |
| | 1 – (P(RR) + P(WW)<br>$1 - \left(\frac{3}{28} + \frac{5}{8} \times \frac{4}{7}\right)$ | **M1** | Method shown, numerical calculations identified, may include replacements |
| | $= \frac{15}{28}$ | **A1** | AG, Fully correct calculations |
| | | **2** | |
| 6(iii) | $P(\text{first red}\mid\text{second red}) = \frac{\textit{their}\ \textbf{(i)}}{\textit{their}\ \textbf{(i)} + \frac{5}{8} \times \frac{3}{7}} = \frac{\frac{3}{8} \times \frac{2}{7}}{\frac{3}{8} \times \frac{2}{7} + \frac{5}{8} \times \frac{3}{7}} = \frac{\frac{3}{28}}{\frac{21}{56}}$ | **M1** | Conditional probability formula used consistent with *their* probabilities or correct |
| | $= \frac{2}{7}$ | **A1** | OE |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(iv) | $x$ \| 0 \| 1 \| 2<br>$p$ \| $\frac{10}{28}$ \| $\frac{15}{28}$ \| $\frac{3}{28}$ | **B1** | Probability distribution table with correct values of $x$ and at least one correct probability placed. Extra $x$ values allowed with probability of zero stated. |
| | | **B1FT** | Fully correct<br>FT P(2) = *their* **(i)**, P(1) = *their* **(ii)**, $\Sigma(p) = 1$. |
| | | **2** | |
| 6(v) | $\mathrm{E}(X) = \frac{30}{56} + \frac{12}{56} = \frac{42}{46} \quad \left(= \frac{3}{4}\right)$ | **B1** | May be implied by use in variance formula |
| | $\mathrm{Var}(X) = \frac{30}{56} + \frac{24}{56} - \left(\textit{their}\ \frac{3}{4}\right)^2$ | **M1** | Substitute into correct variance formula, must have<br>' – *their* mean$^2$'<br>Must be for 2 or more non-zero $x$-values |
| | $\frac{45}{112}$ or 0.402 | **A1** | Correct final answer |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i)(a) | $\mathrm{P}(0, 1, 2) = {}^6\mathrm{C}_0\ 0.3^0\ 0.7^6 + {}^6\mathrm{C}_1\ 0.3^1\ 0.7^5 + {}^6\mathrm{C}_2\ 0.3^2\ 0.7^4$ | **M1** | Binomial term of form ${}^6\mathrm{C}_x p^x (1-p)^{6-x} \quad 0 < p < 1$<br>any $p$, $x \neq 6,0$ |
| | 0.1176 ... + 0.3025 ... + 0.3241 ... | **A1** | Correct unsimplified answer |
| | 0.744 | **A1** | Correct final answer |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(i)(b) | P(support neither choir) = 1 – (0.3 + 0.45) = 0.25 | **M1** | $0.25^n$ seen alone, $1 < n \leqslant 6$ |
| | P(6 support neither choir) = $0.25^6$<br>= 0.000244 or $\frac{1}{4096}$ | **A1** | Correct final answer |
| | | **2** | |
| 7(ii) | Mean = 240 × 0.25 = 60<br>Variance = 240 × 0.25 × 0.75 = 45 | **B1FT** | Correct unsimplified *240p* and *240pq*<br>where p =*their* P(support neither choir) or 0.25 |
| | $P(X < 50) = P\left(Z < \frac{49.5 - 60}{\sqrt{45}}\right) = P(Z < -1.565)$ | **M1** | Substituting *their* $\mu$ and $\sigma$ (condone $\sigma^2$) into the ±Standardisation Formula with a numerical value for *'49.5'*. |
| | | **M1** | Using continuity correction 49.5 or 50.5 within a standardisation expression |
| | 1 – 0.9412 | **M1** | Appropriate area Φ from standardisation formula P(z< …) in final solution, (< 0.5 if *z* is –ve, > 0.5 if *z* is +ve) |
| | 0.0588 | **A1** | Correct final answer |
| | | **5** | |

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