Cambridge International AS & A Level

Cambridge Assessment International Education
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/41**

Paper 4 **October/November 2019**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$20\,000 = V \times 1250g$	**M1**	Use of $P = Fv$ with $F = mg$
	$V = 1.6$	**A1**	
		2	

Question	Answer	Marks	Guidance
2	Initial $KE = \frac{1}{2} \times 75 \times 10^2$ Final $KE = \frac{1}{2} \times 75 \times 5^2$	**B1**	Either correct
	PE gained $= 75g \times 700 \sin 1.5$ [=13 743]	**B1**	
	WD by $F = F \times 700$	**B1**	For WD by $F = F \times d$
	WD by F + Initial KE = Final KE + PE gain + 2000	**M1**	Use of work-energy equation. 5 dimensionally correct terms.
	$F = 18.5$	**A1**	
		5	

Question	Answer	Marks	Guidance
3(i)	$R = 3\,g\cos 60$	**B1**	
	Use $F = \mu R$	**M1**	
	$[3g\sin 60 - \mu 3g\cos 60 - 15 = 0]$	**M1**	Resolve forces parallel to the plane, 3 terms
		A1	Correct equation
	$\mu = 0.732$	**A1**	Allow $\mu = \sqrt{3} - 1$
		5	
3(ii)	[Maximum force $= 3g\sin 60 + F$ $= 3\sin 60 + \mu 3g\cos 60$]	**M1**	
	$X = 37(.0)$	**A1**	Allow $X = 15\left(2\sqrt{3-1}\right)$
		2	

Question	Answer	Marks	Guidance
4(i)	Apply Newton's second law to either or to the system	**M1**	
	Block A: $T - 4g \times \frac{7}{25} = 4a$ Block B: $36 - T - 5g \times \frac{7}{25} = 5a$ System: $36 - 5g \times \frac{7}{25} - 4g \times \frac{7}{25} = 9a$	**A1**	Any two correct. Allow α = 16.3 used.
	Either solving the system for a or solving a pair of simultaneous equations for either a or T	**M1**	
	$a = 1.2$ ms^{-2}	**A1**	
	$T = 16$ N	**A1**	
		5	
4(ii)	$\left[0.65 = 1 \times t + \frac{1}{2} \times 1.2t^2\right]$	**M1**	Use constant acceleration equation(s) with $u = 1$ and solve a 3 term quadratic equation to find t
	$t = 0.5$ s	**A1**	
	Alternative method for question 4(ii)		
	$v^2 = 1^2 + 2 \times 1.2 \times 0.65$ $[v = 1.6]$ and $0.65 = \frac{1}{2}(1 + v) \times t$	**M1**	Use relevant constant acceleration equations with $u = 1$ in a complete method to find t
	$t = 0.5$ s	**A1**	
		2	

Question	Answer	Marks	Guidance
5(i)	Resolve forces either horizontally or vertically	**M1**	
	$7.5\cos60 + 4.5\cos20 = F\cos\theta$ [= 7.97861]	**A1**	
	$7.5\sin60 - 4.5\sin20 = F\sin\theta$ [= 4.95609]	**A1**	
	$F = \sqrt{\left(7.98^2 + 4.96^2\right)}$	**M1**	Use Pythagoras or use the value found for θ to find F
	$\theta = \tan^{-1}\left(\frac{4.96}{7.98}\right)$	**M1**	Use trigonometry or the value found for F to find θ
	$F = 9.39$ and $\theta = 31.8$	**A1**	
	Alternative method for question 5(i)		
	$\frac{F}{\sin 80} = \frac{4.5}{\sin\left(120+\theta\right)} = \frac{7.5}{\sin\left(160-\theta\right)}$	**M1**	Attempt to use Lami
		A1	One correct pair of terms
		A1	A second correct pair of terms
	$[4.5\sin(160 - \theta) = 7.5\sin(120 + \theta)]$	**M1**	Attempt to solve for θ
	Use the θ value found by valid trigonometry to find F	**M1**	
	$F = 9.39$ and $\theta = 31.8$	**A1**	

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Question	Answer	Marks	Guidance
5(i)	**Alternative method for question 5(i)**		
	Forces 4.5, 7.5, F opposite angles $60 - \theta$, $\theta + 20$, 100	**M1**	Illustrate a triangle of forces
	$[F^2 = 4.5^2 + 7.5^2 - 2 \times 4.5 \times 7.5 \times \cos 100]$	**M1**	For application of cosine rule to find F
		A1	Correct equation
	$\left[\frac{9.39}{\sin 100} = \frac{4.5}{\sin(60-\theta)} = \frac{7.5}{\sin(\theta+20)}\right]$	**M1**	One application of the sine rule to find θ
		A1	Correct equation
	$F = 9.39$ and $\theta = 31.8$	**A1**	
		6	
5(ii)	$9.5\cos 30 - 7.5\cos 60 - 4.5\cos 20 = m \times 1.5$	**M1**	Apply Newton's second law to the ring along AB (4 terms)
	$m = 0.166$ kg	**A1**	
		2	

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Question	Answer	Marks	Guidance
6(i)	$0.4g \times 1.8 = \frac{1}{2} \times 0.4 \times v^2$	**M1**	KE gain = PE lost
	$v = 6$ ms^{-1}	**A1**	
	Alternative method for question 6(i)		
	$v^2 = 0^2 + 2 \times g \times 1.8$	**M1**	Use constant acceleration equation(s) with $a = g$ to find v
	$v = 6$ ms^{-1}	**A1**	
		2	
6(ii)	$0.4g - 5.6 = 0.4a$	**M1**	Use Newton's second law for the particle in the vertical (3 terms)
	$a = -4$ ms^{-2}	**A1**	
	$0 = 6 - 4t$	**M1**	Use of constant acceleration equation(s) such as $v = u + at$ to find t
	$t = 1.5$ s	**A1**	
		4	
6(iii)	Straight line starting at (0,0) with positive gradient	**B1**	
	Second straight line starting at end of the first line with negative gradient and ending with $v = 0$	**B1**	
	All correct, start at (0, 0) with max velocity $v = 6$ at $t = 0.6$ i.e. (0.6, 6) and finishing at (2.1, 0)	**B1FT**	FT on *their v* from **(i)** and/or *their t* from **(ii)**
		3	

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Question	Answer	Marks	Guidance
7(i)	$0.6t^2 - 0.12t^3 = 0$	**M1**	For attempting to solve $v = 0$
	$(t = 0$ or$)\ t = 5$	**A1**	
	$\int v\ \mathrm{d}t = 0.2t^3 - 0.03t^4$	***M1**	For integrating the velocity
	$OP = [0.2 \times 5^3 - 0.03 \times 5^4] - [0]$	**DM1**	Use limits to find OP
	Distance = 6.25 m	**A1**	AG
		5	
7(ii)	$k \times 5^3 + c \times 5^5 = 6.25$	**B1**	Using $s = 6.25$ at $t = 5$ to set up equation in k and c
	$v = 3kt^2 + 5ct^4$	***M1**	For differentiating s to find v
	$1.25 = 3k \times 5^2 + 5c \times 5^4$	**DM1**	For using the given value of $v = 1.25$ in the expression for v
	$125k + 3125c = 6.25$ $75k + 3125c = 1.25$	**M1**	For attempting to solve a pair of simultaneous equations in k and c and finding a value of either k or c
	$k = 0.1,\ c = -0.002$	**A1**	
		5	
7(iii)	$a = 0.6t - 0.04t^3$	**M1**	For differentiating their expression for v
	At $t = 5$, $a = -2$ Acceleration $= -2\ \mathrm{ms}^{-2}$	**A1**	
		2	

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