Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/33**

Paper 3 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(x+2)^2 > (3x-1)^2$, or corresponding quadratic equation, or pair of linear equations $2(x + 2) = \pm (3x - 1)$	**B1**	
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	**M1**	
	Obtain critical values $x = -\frac{3}{5}$ and $x = 5$	**A1**	
	State final answer $-\frac{3}{5} < x < 5$	**A1**	
	Alternative method for question 1		
	Obtain critical value $x = 5$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	**B1**	
	Obtain critical value $x = -\frac{3}{5}$ similarly	**B2**	
	State final answer $-\frac{3}{5} < x < 5$	**B1**	
		4	

Question	Answer	Marks	Guidance
2	Substitute $x = -\frac{1}{2}$, equate result to zero and obtain a correct equation, e.g. $-\frac{6}{8}+\frac{1}{4}a-\frac{1}{2}b-2=0$	**B1**	
	Substitute $x = -2$ and equate result to -24	***M1**	
	Obtain a correct equation, e.g. $-48+4a-2b-2=-24$	**A1**	
	Solve for a or for b	**DM1**	
	Obtain $a = 5$ and $b = -3$	**A1**	
		5	

Question	Answer	Marks	Guidance
3	Reduce the equation to a horizontal equation in 3^{3x}, 3^{3x+1} or 27^x	**M1**	
	Simplify and reach $3(3^{3x})=5$, $3\left(27^x\right)=5$, or equivalent	**A1**	
	Use correct method for finding x from a positive value of 3^{3x}, 3^{3x+1} or 27^x	**M1**	
	Obtain answer $x = 0.155$	**A1**	
		4	

Question	Answer	Marks	Guidance
4(i)	Use tan $(A + B)$ formula to express the LHS in terms of tan $2x$ and tan x	**M1**	
	Using the tan $2A$ formula, express the entire equation in terms of tan x	**M1**	
	Obtain a correct equation in tan x in any form	**A1**	
	Obtain the given form correctly	**A1**	AG
		4	
4(ii)	Use correct method to solve the given equation for x	**M1**	
	Obtain answer, e.g. $x = 26.8°$	**A1**	
	Obtain second answer, e.g. $x = 73.7°$ and no other	**A1**	Ignore answers outside the given interval
		3	

Question	Answer	Marks	Guidance
5(i)	Sketch a relevant graph, e.g. $y = \ln(x + 2)$	**B1**	
	Sketch a second relevant graph, e.g. $y = 4e^{-x}$, and justify the given statement	**B1**	Consideration of behaviour for $x < 0$ is needed for the second B1
		2	
5(ii)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 1.5$	**M1**	
	Complete the argument correctly with correct calculated values	**A1**	
		2	

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Question	Answer	Marks	Guidance
5(iii)	Use the iterative formula correctly at least **twice** using output from a previous iteration	**M1**	
	Obtain final answer 1.23	**A1**	
	Show sufficient iterations to 4 d.p. to justify 1.23 to 2 d.p., or show there is a sign change in the interval (1.225, 1.235)	**A1**	
		3	

Question	Answer	Marks	Guidance
6(i)	Obtain answer $w=\frac{1}{2}+\frac{\sqrt{3}}{2}\mathrm{i}$	**B1**	
		1	
6(ii)	Show point representing u	**B1**	
	Show point representing v in relatively correct position	**B1**	
		2	
6(iii)	Explain why the moduli are equal	**B1**	
	Explain why the arguments are equal	**B1**	
	Use $\mathrm{i}^2=-1$ and obtain $2uw$ in the given form	**M1**	
	Obtain answer $1-2\sqrt{3}+\left(2+\sqrt{3}\right)\mathrm{i}$	**A1**	
		4	

Question	Answer	Marks	Guidance
7(i)	Substitute coordinates (5, 2, – 2) in $x + 4y – 8z = d$	**M1**	
	Obtain plane equation $x + 4y – 8z = 29$, or equivalent	**A1**	
		2	
7(ii)	Attempt to use perpendicular formula to find perpendicular from (5, 2, – 2) to m	**M1**	
	Obtain a correct unsimplified expression, e.g. $\frac{5+8+16-2}{\sqrt{(1+16+64)}}$	**A1**	
	Obtain answer 3	**A1**	
	Alternative method 1 for question 7(ii)		
	State or imply perpendicular from O to m is $\frac{2}{9}$ or from O to n is $\frac{29}{9}$	**B1**	
	Find difference in perpendiculars	**M1**	
	Obtain answer 3	**A1**	
	Alternative method 2 for question 7(ii)		
	Obtain correct parameter value, or position vector or coordinates of the foot of the perpendicular from (5, 2, – 2) to m, e.g. $\mu = \pm\frac{1}{3}$; $\left(\frac{14}{3}, \frac{2}{3}, \frac{2}{3}\right)$	**B1**	
	Calculate the length of the perpendicular	**M1**	
	Obtain answer 3	**B1**	
		3	

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Question	Answer	Marks	Guidance
7(iii)	Calling the direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, use a scalar product to form a relevant equation in a, b and c, e.g. $a + 4b - 8c = 0$ or $5a + 2b - 2z = 0$	**B1**	
	Solve two relevant equations for the ratio $a : b : c$	**M1**	
	Obtain $a : b : c = 4 : -19 : -9$	**A1**	OE
	State answer $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} - 19\mathbf{j} - 9\mathbf{k})$	**A1**	OE
	Alternative method for question 7(iii)		
	Attempt to calculate vector product of two relevant vectors, e.g. $(\mathbf{i} + 4\mathbf{j} - 8\mathbf{k})\times(5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$	**M1**	
	Obtain two correct components	**A1**	
	Obtain $8\mathbf{i} - 38\mathbf{j} - 18\mathbf{k}$	**A1**	OE
	State answer $\mathbf{r} = 5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k} + \lambda(4\mathbf{i} - 19\mathbf{j} - 9\mathbf{k})$	**A1**	OE
		4	

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Question	Answer	Marks	Guidance
8(i)	State or imply ordinates 1, 1.2116…, 2.7597...	**B1**	
	Use correct formula, or equivalent, with $h = 0.6$	**M1**	
	Obtain answer 1.85	**A1**	
		3	
8(ii)	Explain why the rule gives an overestimate	**B1**	
		1	
8(iii)	Differentiate using quotient or chain rule	**M1**	
	Obtain correct derivative in terms of $\sin x$ and $\cos x$	**A1**	
	Equate derivative to 2, use Pythagoras and obtain an equation in $\sin x$	**M1**	
	Obtain $2\sin^2 x + \sin x - 2 = 0$	**A1**	OE
	Solve a 3-term quadratic for x	**M1**	
	Obtain answer $x = 0.896$ only	**A1**	
		6	

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Question	Answer	Marks	Guidance
9(i)	Separate variables correctly and integrate one side	**B1**	
	Obtain term $0.2t$, or equivalent	**B1**	
	Carry out a relevant method to obtain A and B such that $\frac{1}{(20-x)(40-x)} \equiv \frac{A}{20-x} + \frac{B}{40-x}$	***M1**	OE
	Obtain $A = \frac{1}{20}$ and $B = -\frac{1}{20}$	**A1**	
	Integrate and obtain terms $-\frac{1}{20}\ln(20-x) + \frac{1}{20}\ln(40-x)$ OE	**A1FT +A1FT**	The FT is on A and B
	Use $x = 10$, $t = 0$ to evaluate a constant, or as limits	**DM1**	
	Obtain correct answer in any form	**A1**	
	Obtain final answer $x = \frac{60e^{4t} - 40}{3e^{4t} - 1}$	**A1**	OE
		9	
9(ii)	State that x approaches 20	**B1**	
		1	

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Question	Answer	Marks	Guidance
10(i)	Use product rule and chain rule at least once	**M1**	
	Obtain correct derivative in any form	**A1**	
	Equate derivative to zero, use Pythagoras and obtain an equation in cos x	**M1**	
	Obtain $\cos^2 x + 3\cos x - 1 = 0$, or 3-term equivalent	**A1**	
	Obtain answer $x = 1.26$	**A1**	
		5	
10(ii)	Using $\mathrm{d}u = \pm \sin x \, \mathrm{d}x$ express integrand in terms of u and $\mathrm{d}u$	**M1**	
	Obtain integrand $\mathrm{e}^u\left(u^2 - 1\right)$	**A1**	OE
	Commence integration by parts and reach $a\mathrm{e}^u\left(u^2 - 1\right) + b\int u\mathrm{e}^u \, \mathrm{d}u$	***M1**	
	Obtain $\mathrm{e}^u\left(u^2 - 1\right) - 2\int u\mathrm{e}^u \, \mathrm{d}u$	**A1**	OE
	Complete integration, obtaining $\mathrm{e}^u\left(u^2 - 2u + 1\right)$	**A1**	OE
	Substitute limits $u = 1$ and $u = -1$ (or $x = 0$ and $x = \pi$), having integrated completely	**DM1**	
	Obtain answer $\frac{4}{\mathrm{e}}$, or exact equivalent	**A1**	
		7	