Cambridge International AS & A Level

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MATHEMATICS **9709/13**

Paper 1 **October/November 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(i)	$1 + 6y + 15y^2$	**B1**	CAO
		1	
1(ii)	$1+6\left(px-2x^2\right)+15\left(px-2x^2\right)^2$	**M1**	SOI. Allow $6C1\times1^5\left(px-2x^2\right)$, $6C2\times1^4\left(px-2x^2\right)^2$
	$\left(15p^2-12\right)(x^2)=48\left(x^2\right)$	**A1**	1 term from each bracket and equate to 48
	$p=2$	**A1**	**SC:** A1 $p = 4$ from $15p - 12 = 48$
		3	

Question	Answer	Marks	Guidance
2	$(y=)\left[\left(x-3\right)^2\right][-2]$	***B1** **DB1**	DB1 dependent on 3 in 1st bracket
	$x-3=(\pm)\sqrt{y+2}$ **or** $y-3=(\pm)\sqrt{x+2}$	**M1**	Correct order of operations
	$\left(\mathrm{g}^{-1}(x)\right)=3+\sqrt{x+2}$	**A1**	Must be in terms of x
	Domain (of g^{-1}) is $(x) > -1$	**B1**	Allow $(-1, \infty)$. Do not allow $y > -1$ or $\mathrm{g}(x) > -1$ or $\mathrm{g}^{-1}(x) > -1$
		5	

Question	Answer	Marks	Guidance
3	$\frac{dy}{dx} = 3x^2 + 2x - 8$	**B1**	
	Set to zero (SOI) and solve	**M1**	
	(Min) $a = -2$, (Max) $b = 4/3$. – in terms of a and b.	**A1** **A1**	Accept $a \geqslant -2$, $b \leqslant \frac{4}{3}$ **SC**: A1 for $a > -2$, $b < \frac{4}{3}$ or for $-2 < x < \frac{4}{3}$
		4	

Question	Answer	Marks	Guidance
4(i)	Angle $CAO = \frac{\pi}{3}$	**B1**	
		1	
4(ii)	(Sector AOC) $= \frac{1}{2}r^2 \times their \frac{\pi}{3}$	**M1**	SOI
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(their \frac{\pi}{3}\right)$ **or** $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ **or** $\frac{1}{2}(r)(r)\sqrt{3}$	**M1**	For M1M1, $their \frac{\pi}{3}$ must be of the form $k\pi$ where $0 < k < ½$
	$(\Delta ABC) = \frac{1}{2}(r)(2r)\sin\left(\frac{\pi}{3}\right)$ **or** $\frac{1}{2}(2r)(r)\frac{\sqrt{3}}{2}$ **or** $\frac{1}{2}(r)(r)\sqrt{3}$	**A1**	All correct
	$r^2\left(\frac{\sqrt{3}}{2}\right) - \frac{1}{2}r^2\left(\frac{\pi}{3}\right)$	**A1**	
		4	

Question	Answer	Marks	Guidance
5(i)	$S = 28x^2$, $V = 8x^3$	**B1B1**	SOI
	$7V^{\frac{2}{3}} = 7 \times 4x^2 = S$	**B1**	AG, WWW
		3	
5(ii)	$\left(\frac{\mathrm{d}S}{\mathrm{d}V}\right) = \frac{14V^{-\frac{1}{3}}}{3} = \frac{14}{30}$ SOI when $V = 1000$	***M1** **A1**	Attempt to differentiate For M mark $\left(\frac{\mathrm{d}S}{\mathrm{d}V}\right)$ to be of form $kV^{-\frac{1}{3}}$
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}t} \times \frac{\mathrm{d}V}{\mathrm{d}S}\right)$ OE used with $\frac{\mathrm{d}S}{\mathrm{d}t} = 2$ **and** $\frac{1}{their\ \frac{14}{30}}$	**DM1**	
	$\frac{30}{7}$ or 4.29	**A1**	OE
	Alternative method for question 5(ii)		
	$V = \frac{S^{\frac{3}{2}}}{7\sqrt{7}} \rightarrow \left(\frac{\mathrm{d}V}{\mathrm{d}S}\right) = \frac{3}{2} \times S^{\frac{1}{2}} \times \frac{1}{7\sqrt{7}} = \frac{30}{14}$ SOI when $S = 700$	***M1** **A1**	Attempt to differentiate For M mark $\left(\frac{\mathrm{d}V}{\mathrm{d}S}\right)$ to be of form $kS^{\frac{1}{2}}$
	$\left(\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}S}{\mathrm{d}t} \times \frac{\mathrm{d}V}{\mathrm{d}S}\right)$ OE used with $\frac{\mathrm{d}S}{\mathrm{d}t} = 2$ **and** $\frac{1}{their\ \frac{14}{30}}$	**DM1**	
	$\frac{30}{7}$ or 4.29	**A1**	OE

Question	Answer	Marks	Guidance
5(ii)	**Alternative method for question 5(ii)**		
	Attempt to find either $\frac{\mathrm{d}V}{\mathrm{d}x}$ **or** $\left(\frac{\mathrm{d}S}{\mathrm{d}x} \text{ and } \frac{\mathrm{d}V}{\mathrm{d}S}\right)$ together with either $\frac{\mathrm{d}x}{\mathrm{d}t}$ **or** x	***M1**	
	$\frac{\mathrm{d}V}{\mathrm{d}x} = 24x^2$ **or** $\left(\frac{\mathrm{d}S}{\mathrm{d}x} = 56x \text{ and } \frac{\mathrm{d}V}{\mathrm{d}S} = \frac{3x}{7}\right)$, $\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{140}$ **or** $x = 5$ (A1)	**A1**	
	Correct method for $\frac{\mathrm{d}V}{\mathrm{d}t}$	**DM1**	
	$\frac{30}{7}$ or 4.29	**A1**	OE
		4	

Question	Answer	Marks	Guidance
6(i)	$3kx - 2k = x^2 - kx + 2 \rightarrow\ x^2 - 4kx + 2k + 2\ (=0)$	**B1**	kx terms combined correctly-implied by correct $b^2 - 4ac$
	Attempt to find $b^2 - 4ac$	**M1**	Form a quadratic equation in k
	1 and $-\frac{1}{2}$	**A1**	SOI
	$k > 1,\ k < -\frac{1}{2}$	**A1**	Allow $x > 1,\ x < -1/2$
		4	
6(ii)	$y = 3x - 2, \quad y = -\frac{3}{2}x + 1$	**M1**	Use of *their* k values (twice) in $y = 3kx - 2k$
	$3x - 2 = -\frac{3}{2}x + 1$ OR $y + 2 = 2 - 2y$	**M1**	Equate *their* tangent equations OR substitute $y = 0$ into both lines
	$x = \frac{2}{3}$, $\rightarrow y = 0$ in one or both lines	**A1**	Substitute $x = \frac{2}{3}$ in one or both lines
		3	

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Question	Answer	Marks	Guidance
7(i)	$3\cos^4\theta + 4\left(1-\cos^2\theta\right) - 3\left(=0\right)$	**M1**	Use $s^2 = 1 - c^2$
	$3x^2 + 4\left(1-x\right) - 3\left(=0\right) \rightarrow\ 3x^2 - 4x + 1\left(=0\right)$	**A1**	AG
		2	
7(ii)	Attempt to solve for x	**M1**	Expect x = 1, 1/3
	$\cos\theta = \left(\pm\right)1,\ \left(\pm\right)0.5774$	**A1**	Accept $\left(\pm\right)\left(\frac{1}{\sqrt{3}}\right)$ SOI
	(θ =) 0º, 180º, 54.7º, 125.3º	**A3,2,1,0**	A2,1,0 if more than 4 solutions in range
		5	

Question	Answer	Marks	Guidance
8(i)	$(2x-1)^{\frac{1}{2}} < 2$ or $3(2x-1)^{\frac{1}{2}} < 6$	**M1**	SOI
	$2x-1<4$	**A1**	SOI
	$\frac{1}{2} < x < \frac{5}{2}$	**A1 A1**	Allow 2 separate statements
		4	
8(ii)	$f(x) = [3(2x-1)^{3/2} \div \left(\frac{3}{2}\right) \div (2)]$ $[-6x]$ (+c)	**B1 B1**	
	Subsitute $x = 1$, $y = -3$ into an integrated expression.	**M1**	Dependent on c being present ($c = 2$)
	$f(x) = (2x-1)^{\frac{3}{2}} - 6x + 2$	**A1**	
		4	

Question	Answer	Marks	Guidance
9(i)	$\frac{5k-6}{3k}=\frac{6k-4}{5k-6} \quad \rightarrow \quad (5k-6)^2=3k(6k-4)$	**M1**	OR any valid relationship
	$25k^2-60k+36=18k^2-12k \quad \rightarrow \quad 7k^2-48k+36$	**A1**	AG
		2	
9(ii)	$k=\frac{6}{7}$, 6	**B1B1**	Allow 0.857(1) for $\frac{6}{7}$
	When $k=\frac{6}{7}$, $r=-\frac{2}{3}$	**B1**	Must be exact
	When $k=6$, $r=\frac{4}{3}$	**B1**	
		4	
9(iii)	Use of $S_\infty=\frac{a}{1-r}$ with $r=their-\frac{2}{3}$ and $a=3\times their\frac{6}{7}$	**M1**	Provided $0 < \lvert their -2/3\rvert < 1$
	$\frac{18}{7}\div\left(1+\frac{2}{3}\right)=\frac{54}{35}$ or 1.54	**A1**	FT if 0.857(1) has been used in part (ii).
		2	

Question	Answer	Marks	Guidance
10(i)	$\mathbf{AX} = \begin{pmatrix} 6 \\ 2 \\ 3 \end{pmatrix}$, and one of $\mathbf{AB} = \begin{pmatrix} 18 \\ 6 \\ 9 \end{pmatrix}$, $\mathbf{XB} = \begin{pmatrix} 12 \\ 4 \\ 6 \end{pmatrix}$, $\mathbf{BX} = \begin{pmatrix} -12 \\ -4 \\ -6 \end{pmatrix}$	**B1B1**	
	State $\mathbf{AB} = 3\mathbf{AX}$ (or $\mathbf{XB} = 2\mathbf{AX}$ or $\mathbf{AB} = \frac{3}{2}\mathbf{XB}$ etc) hence straight line **OR** $\frac{\mathbf{AX}.\mathbf{AB}}{\lvert\mathbf{AX}\rvert\lvert\mathbf{AB}\rvert} = 1 \ (\rightarrow \theta = 0)$ or $\frac{\mathbf{AX}.\mathbf{BX}}{\lvert\mathbf{AX}\rvert\lvert\mathbf{BX}\rvert} = -1 \ (\rightarrow \theta = 180)$ hence straight line	**B1**	WWW A conclusion (i.e. a straight line) is required.
		3	
10(ii)	$\mathbf{CX} = \begin{pmatrix} -3 \\ 6 \\ 2 \end{pmatrix}$	**B1**	
	$\mathbf{CX}.\mathbf{AX} = -18 + 12 + 6$	**M1**	
	= 0 (hence *CX* is perpendicular to *AX*)	**A1**	
		3	
10(iii)	$\lvert\mathbf{CX}\rvert = \sqrt{3^2 + 6^2 + 2^2}$, $\lvert\mathbf{AB}\rvert = \sqrt{18^2 + 6^2 + 9^2}$ Both attempted	**M1**	
	Area $\Delta ABC = \frac{1}{2} \times$ *their* $21 \times$ *their* $7 = 73\frac{1}{2}$	**M1A1**	Accept answers which round to 73.5
		3	

Question	Answer	Marks	Guidance
11(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = -2(x-1)^{-3}$	**B1**	
	When $x = 2$, $m = -2 \rightarrow$ gradient of normal $= -\frac{1}{m}$	**M1**	m must come from differentiation
	Equation of normal is $y - 3 = \frac{1}{2}(x-2) \rightarrow \quad y = \frac{1}{2}x + 2$	**A1**	AG Through (2, 3) with gradient $-\frac{1}{m}$. Simplify to AG
		3	

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Question	Answer	Marks	Guidance
11(ii)	$(\pi)\int y_1^2\,(\mathrm{d}x), (\pi)\int y_2^2\,(\mathrm{d}x)$	***M1**	Attempt to integrate y^2 for at least one of the functions
	$(\pi)\int\left(\frac{1}{2}x+2\right)^2$ or $\left(\frac{1}{4}x^2+2x+4\right)$ $(\pi)\int\left((x-1)^{-4}+4(x-1)^{-2}+4\right)$	**A1A1**	A1 for $\left(\frac{1}{2}x+2\right)^2$ depends on an attempt to integrate this form later
	$(\pi)\left[\frac{2}{3}\left(\frac{1}{2}x+2\right)^3\right.$ or $\left.\frac{1}{12}x^3+x^2+4x\right]$ $(\pi)\left[\frac{(x-1)^{-3}}{-3}+\frac{4(x-1)^{-1}}{-1}+4x\right]$	**A1A1**	Must have at least 2 terms correct for each integral
	$(\pi)\left\{18-\frac{125}{12}\,or\,\frac{2}{3}+4+8-\left(\frac{1}{12}+1+4\right)\right\}\ \left\{\frac{-1}{24}-2+12-\left(\frac{-1}{3}-4+8\right)\right\}$	**DM1**	Apply limits to at least 1 integrated expansion
	Attempt to add 2 volume integrals (or 1 volume integral + frustum) $\pi\left\{7\frac{7}{12}+6\frac{7}{24}\right\}$	**DM1**	
	$13\frac{7}{8}\pi$ or $\frac{111}{8}\pi$ or 13.9π or 43.6	**A1**	$\frac{2}{3}+4+8-\left(\frac{1}{12}+1+4\right)$ $\frac{-1}{24}-2+12-\left(\frac{-1}{3}-4+8\right)$
		8	