Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2016**

1 hour 15 minutes

Additional Materials: List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.

This document consists of **3** printed pages, **1** blank page and **1** insert.

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1 The weights, in kilograms, of a random sample of eight 16-year old males are given below.

58.9 63.5 62.7 59.4 66.9 68.0 60.4 68.2

Find unbiased estimates of the population mean and variance of the weights of all 16-year old males. [3]

2 A die has six faces numbered 1, 2, 3, 4, 5, 6. Manjit suspects that the die is biased so that it shows a six on fewer throws than it would if it were fair. In order to test her suspicion, she throws the die a certain number of times and counts the number of sixes.

(i) State suitable null and alternative hypotheses for Manjit's test. [1]

(ii) There are no sixes in the first 15 throws. Show that this result is not significant at the 5% level. [2]

(iii) Find the smallest value of n such that, if there are no sixes in the first n throws, this result is significant at the 5% level. [2]

3 Particles are emitted randomly from a radioactive substance at a constant average rate of 3.6 per minute. Find the probability that

(i) more than 3 particles are emitted during a 20-second period, [3]

(ii) more than 240 particles are emitted during a 1-hour period. [4]

4 Each week a farmer sells X litres of milk and Y kg of cheese, where X and Y have the independent distributions $N(1520, 53^2)$ and $N(175, 12^2)$ respectively.

(i) Find the mean and standard deviation of the total amount of milk that the farmer sells in 4 randomly chosen weeks. [2]

During a year when milk prices are low, the farmer makes a loss of 2 cents per litre on milk and makes a profit of 21 cents per kg on cheese, so the farmer's overall weekly profit is $(21Y - 2X)$ cents.

(ii) Find the probability that, in a randomly chosen week, the farmer's overall profit is positive. [5]

5 **(a)** The masses, in grams, of certain tomatoes are normally distributed with standard deviation 9 grams. A random sample of 100 tomatoes has a sample mean of 63 grams. Find a 90% confidence interval for the population mean mass of these tomatoes. [3]

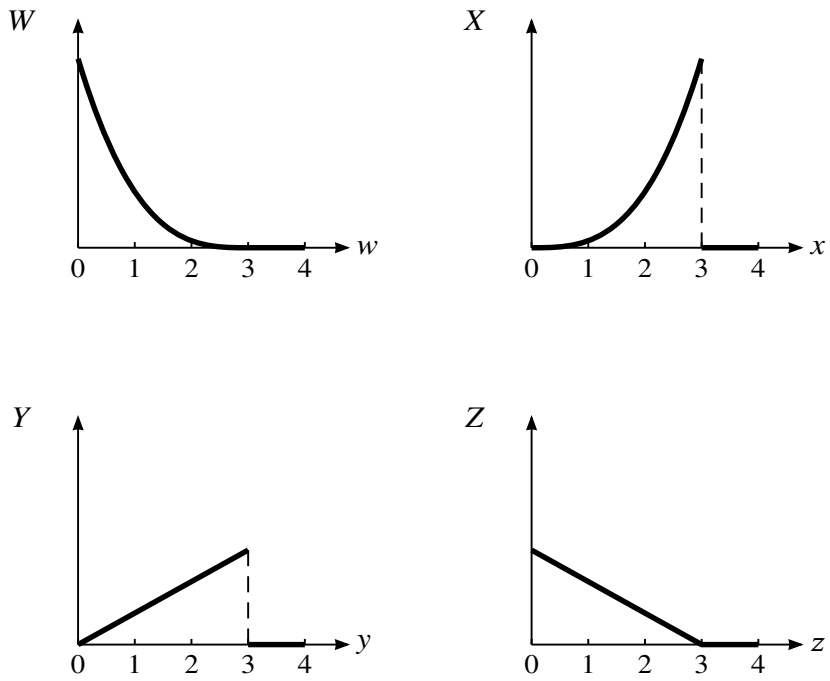
(b) The masses, in grams, of certain potatoes are normally distributed with known population standard deviation but unknown population mean. A random sample of potatoes is taken in order to find a confidence interval for the population mean. Using a sample of size 50, a 95% confidence interval is found to have width 8 grams.

(i) Using another sample of size 50, an α% confidence interval has width 4 grams. Find α. [3]

(ii) Find the sample size n, such that a 95% confidence interval has width 4 grams. [2]

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The diagrams show the probability density functions of four random variables W, X, Y and Z. Each of the four variables takes values between 0 and 3 only, and their medians are m_W, m_X, m_Y and m_Z respectively.

(i) List m_W, m_X, m_Y and m_Z in order of size, starting with the largest. [2]

(ii) The probability density function of X is given by

$$\mathrm{f}(x) = \begin{cases} \frac{4}{81}x^3 & 0 \leqslant x \leqslant 3, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Show that $\mathrm{E}(X) = \frac{12}{5}$. [3]

(b) Calculate $\mathrm{P}\big(X > \mathrm{E}(X)\big)$. [3]

(c) Write down the value of $\mathrm{P}\big(X < 2\mathrm{E}(X)\big)$. [1]

7 In the past the time, in minutes, taken for a particular rail journey has been found to have mean 20.5 and standard deviation 1.2. Some new railway signals are installed. In order to test whether the mean time has decreased, a random sample of 100 times for this journey are noted. The sample mean is found to be 20.3 minutes. You should assume that the standard deviation is unchanged.

(i) Carry out a significance test, at the 4% level, of whether the population mean time has decreased. [5]

Later another significance test of the same hypotheses, using another random sample of size 100, is carried out at the 4% level.

(ii) Given that the population mean is now 20.1, find the probability of a Type II error. [5]

(iii) State what is meant by a Type II error in this context. [1]

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