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MATHEMATICS **9709/41**

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MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1	$[0.4g - T = 0.4a \quad T = 0.6a$ System equation $\quad 0.4g = (0.4 + 0.6)a]$	**M1**		For applying Newton's 2nd law to either particle or to the system
		M1		For applying Newton's 2nd law to the other particle and attempt to solve for a and T
	$a = 4\,\text{m s}^{-2}$	**A1**		
	$T = 2.4\,\text{N}$	**A1**	[4]	
2 **(i)**	$2 = 5a \rightarrow a = 0.4\ \text{ms}^{-2}$	**B1**		
	$[0.1g \sin 20 - F = 0.1 \times 0.4]$	**M1**		For applying Newton's 2nd law to the particle
	$F = 0.302\,\text{N}$ AG	**A1**	[3]	
(ii)	$[R = 0.1g \cos 20 \ (= 0.9397)]$	**M1**		For attempting to find R and using $\mu = F/R$
	$\mu = 0.3020/0.9397 = 0.321$	**A1**	[2]	
3 **(i)**	$[0 = 6^2 - 2g \times s]$	**M1**		For using $v^2 = u^2 + 2as$
	$s = 1.8$	**A1**		
	Total height = 2.3 m	**B1**	[3]	
	Alternative for 3(i)			
	$[6^2 = u^2 - 2g \times 0.5]$	**M1**		For using $v^2 = u^2 + 2as$ to find the initial velocity
	$u^2 = 46$	**A1**		
	$0^2 = 46 - 2gs \rightarrow s$ = total height = 2.3 m	**B1**	[3]	

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(ii)	$[2.3 = 0 + 0.5gt^2]$	**M1**		For using $s = ut + 0.5gt^2$ to find time to reach the ground
	$t = 0.678$	**A1**		
	Total time = $2 \times 0.678 = 1.36\,\text{s}$	**B1**	[3]	
	Alternative for 3(ii)			
	$[0 = \sqrt{46} - gt]$	**M1**		Using $v = u - gt$ to find time taken to the highest point
	$t = \frac{\sqrt{46}}{10} = 0.678$	**A1**		
	Total time = $2 \times 0.678 = 1.36\,\text{s}$	**B1**	[3]	
4		**M1**		For resolving forces horizontally
	$2F + F\cos 60 = 15\cos\alpha$	**A1**		
		M1		For resolving forces vertically
	$F\sin 60 = 15\sin\alpha$	**A1**		
		M1		For using Pythagoras or for using tan α to find F **and** α
	$F = 5.67$ and $\alpha = 19.1$	**A1**	[6]	Allow $F = 15\sqrt{7}/7$
5 (i)	$a = 0.5\,\text{m s}^{-2}$	**B1**	[1]	
(ii)	[Distance $= 25 + 100 + 5(5 + V) + 30V + 10V$]	**M1**		For attempting to find the distance travelled
	$150 + 45V$ AG	**A1**		
	$150 + 45V = 465 \rightarrow V = 7\,\text{m s}^{-1}$	**B1**	[3]	
(iii)	$\frac{1}{2} \times 80 \times 7^2 - \frac{1}{2} \times 80 \times 5^2$ [= 960]	**M1**		For change in KE
	$20 \times (5 + 7)/2 \times 10$ [=1200]	**M1**		For work done against friction using $F \times d$
	$[80gh = 960 + 1200]$	**M1**		For using PE loss = KE gain + WD against Res.
	$h = 2.7\,\text{m}$	**A1**	[4]	

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6 (i)	[Work done = 50 cos10 × 20]	**M1**		Using WD = $Fd\cos\theta$
	= 984.8 J	**A1**	[2]	
(ii)	[984.8 = ½ × 25v^2 + 30 × 20]	**M1**		Using WD by DF = KE gain + WD against Res.
	v = 5.55 m s^{-1}	**A1**	[2]	
(iii)		**M1**		For using Power = Fv
	Max power = 50cos10 × 5.55 = 273 W	**A1**	[2]	Greatest power is at v_{max}
(iv)	[50 cos10 – 30 – 25g sin5 = 25a]	**M1**		For using Newton's 2nd law up the plane
	a = – 0.102 m s^{-2}	**A1**		
	[0 = 5.55 – 0.102t]	**M1**		For using $v = u + at$
	Time t = 54.4 s	**A1**	[4]	
	Alternative for 6(iv)			
		M1		For using WD by DF + KE loss = PE gain + WD against Res to find distance s up plane
	50 cos 10 × s + ½ × 25 × 5.55^2 = 25g × s sin 5 + 30 × s	**A1**		s = 151 m
		M1		For using $s = ½(u + v)t$
	t = 302/5.55 = 54.4 s	**A1**	[4]	
7 (i)	[15 – 6t = 0]	**M1**		For differentiation
	Max acceleration when t = 2.5 s	**A1**		May be stated from an a-t diagram
	Max acceleration = 18.75 m s^{-2}	**A1**	[3]	
(ii)	[Speed = 7.5t^2 – t^3 (+ c)]	**M1**		For using integration to obtain speed
	[Distance = 2.5t^3 – 0.25t^4 (+ ct + d)]	**M1**		For using integration to obtain distance
	= 2.5×125 – 0.25×625 = 156.25 m	**A1**	[3]	Allow distance = 625/4

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(iii)	$v(5) = 7.5\times25 - 125 = 62.5\,\text{m s}^{-1}$	**B1**		Allow $v(5) = 125/2$
	$\int_5^k -\frac{625}{t^2}\,\text{d}t = \left[\frac{625}{t}\right]_5^k$	**M1**		Integral with correct limits
	$= \frac{625}{k} - \frac{625}{5} = \frac{625}{k} - 125$	**A1**		
	$\frac{625}{k} - 125 = v(k) - v(5) = -62.5$	**M1**		Use of $v(5) = 62.5$ and $v(k) = 0$
	$k = 10$	**A1**	[5]	
	Alternative for 7(iii)			
	$v(5) = 7.5\times25 - 125 = 62.5\,\text{m s}^{-1}$	**B1**		
	$v(t) = \int -\frac{625}{t^2}\,\text{d}t = \frac{625}{t} + c$	**M1**		Using indefinite integration
	$[c = -62.5]$ $v(t) = \frac{625}{t} - 62.5$	**A1**		For using $v(5) = 62.5$ to find c and setting $v(k) = 0$
	$v(k) = \frac{625}{k} - 62.5 = 0$	**M1**		
	$k = 10$	**A1**	[5]	

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