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MATHEMATICS **9709/31**

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MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1	Solve for 3^x and obtain $3^x = \frac{18}{7}$ Use correct method for solving an equation of the form $3^x = a$, where $a > 0$ Obtain answer $x = 0.860$ 3 d.p. only	**B1** **M1** **A1**	[3]
2	State correct unsimplified first two terms of the expansion of $(1+2x)^{-\frac{3}{2}}$, e.g. $1+(-\frac{3}{2})(2x)$ State correct unsimplified term in x^2, e.g. $(-\frac{3}{2})(-\frac{3}{2}-1)(2x)^2 / 2!$ Obtain sufficient terms of the product of $(2 – x)$ and the expansion up to the term in x^2 Obtain **final** answer $2-7x+18x^2$ Do not ISW	**B1** **B1** **M1** **A1**	[4]
3	*EITHER*: Correctly restate the equation in terms of $\sin\theta$ and $\cos\theta$ Correct method to obtain a horizontal equation in $\sin\theta$ Reduce the equation to a correct quadratic in any form, e.g. $3\sin^2\theta - \sin\theta - 2 = 0$ Solve a three-term quadratic for $\sin\theta$ Obtain final answer $\theta = –41.8°$ only [Ignore answers outside the given interval.] *OR* 1: Square both sides of the equation and use $1 + \tan^2\theta = \sec^2\theta$ Correct method to obtain a horizontal equation in $\sin\theta$ Reduce the equation to a correct quadratic in any form, e.g. $9\sin^2\theta - 6\sin\theta - 8 = 0$ Solve a three-term quadratic for $\sin\theta$ Obtain final answer $\theta = –41.8°$ only *OR* 2: Multiply through by $(\sec\theta + \tan\theta)$ Use $\sec^2\theta – \tan^2\theta = 1$ Obtain $1 = 3 + 3\sin\theta$ Solve for $\sin\theta$ Obtain final answer $\theta = –41.8°$ only	**B1** **M1** **A1** **M1** **A1** **B1** **M1** **A1** **M1** **A1** **M1** **B1** **A1** **M1** **A1**	[5]

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4	*EITHER*:	*EITHER*:	State $2xy + x^2\frac{dy}{dx}$, or equivalent, as derivative of x^2y	**B1**
			State $6y^2 + 12xy\frac{dy}{dx}$, or equivalent, as derivative of $6xy^2$	**B1**
		OR:	Differentiating LHS using correct product rule, state term $xy(1-6\frac{dy}{dx})$, or equivalent	**B1**
			State term $(y + x\frac{dy}{dx})(x - 6y)$, or equivalent	**B1**
			Equate attempted derivative of LHS to zero and set $\frac{dy}{dx}$ equal to zero	**M1***
			Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	**A1**
			Explicitly reject $y = 0$ as a possibility $py^2 - qxy = 0$	**A1**
			Obtain an equation in x or y	**DM1**
			Obtain answer $(-3a, -a)$	**A1**
	OR:		Rearrange to $y = \frac{9a^3}{x(x-6y)}$ and use correct quotient rule to obtain $-\frac{9a^3}{x^2(x-6y)^2} \times .....$	**B1**
			State term $(x-6y) + x(1-6y')$, or equivalent	**B1**
			Justify division by $x(x - 6y)$	**B1**
			Set $\frac{dy}{dx}$ equal to zero	**M1***
			Obtain a horizontal equation, e.g. $6y^2 - 2xy = 0$ (from correct work only)	**A1**
			Obtain an equation in x or y	**DM1**
			Obtain answer $(-3a, -a)$	**A1** [7]
5 **(i)**	*EITHER*:		Use tan 2*A* formula to express LHS in terms of $\tan\theta$	**M1**
			Express as a single fraction in any correct form	**A1**
			Use Pythagoras or cos 2*A* formula	**M1**
			Obtain the given result correctly	**A1**
	OR:		Express LHS in terms of $\sin 2\theta$, $\cos 2\theta$, $\sin\theta$ and $\cos\theta$	**M1**
			Express as a single fraction in any correct form	**A1**
			Use Pythagoras or cos 2*A* formula or sin(*A* – *B*) formula	**M1**
			Obtain the given result correctly	**A1** [4]
(ii)	Integrate and obtain a term of the form $a\ln(\cos 2\theta)$ or $b\ln(\cos\theta)$ (or secant equivalents)			**M1***
	Obtain integral $-\frac{1}{2}\ln(\cos 2\theta) + \ln(\cos\theta)$, or equivalent			**A1**
	Substitute limits correctly (expect to see use of <u>both</u> limits)			**DM1**
	Obtain the given answer following full and correct working			**A1** [4]

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6 (i)	Make recognizable sketch of a relevant graph Sketch the other relevant graph and justify the given statement	**B1** **B1**	[2]
(ii)	Use calculations to consider the value of a relevant expression at $x = 1.4$ and $x = 1.6$, or the values of relevant expressions at $x = 1.4$ and $x = 1.6$ Complete the argument correctly with correct calculated values	**M1** **A1**	[2]
(iii)	State $x = 2\sin^{-1}\left(\frac{3}{x+3}\right)$ Rearrange this in the form $\text{cosec}\,\frac{1}{2}x = \frac{1}{3}x + 1$ If working in reverse, need $\sin\frac{x}{2} = \left(\frac{3}{x+3}\right)$ for first B1	**B1** **B1**	[2]
(iv)	Use the iterative formula correctly at least once Obtain final answer 1.471 Show sufficient iterations to 5 d.p. to justify 1.471 to 3 d.p., or show there is a sign change in the interval (1.4705, 1.4715)	**M1** **A1** **A1**	[3]
7 (i)	Use the correct product rule Obtain correct derivative in any form, e.g. $(2-2x)e^{\frac{1}{2}x} + \frac{1}{2}(2x-x^2)e^{\frac{1}{2}x}$ Equate derivative to zero and solve for x Obtain $x = \sqrt{5} - 1$ only	**M1** **A1** **M1** **A1**	[4]
(ii)	Integrate by parts and reach $a(2x-x^2)e^{\frac{1}{2}x} + b\int(2-2x)e^{\frac{1}{2}x}\,dx$ Obtain $2e^{\frac{1}{2}x}(2x-x^2) - 2\int(2-2x)e^{\frac{1}{2}x}dx$, or equivalent Complete the integration correctly, obtaining $(12x - 2x^2 - 24)e^{\frac{1}{2}x}$, or equivalent Use limits $x = 0$, $x = 2$ correctly having integrated by parts twice Obtain answer $24 - 8e$, or <u>exact</u> simplified equivalent	**M1*** **A1** **A1** **DM1** **A1**	[5]

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8	**(i)**	State or imply a correct normal vector to either plane, e.g. $3\mathbf{i}+\mathbf{j}-\mathbf{k}$ or $\mathbf{i}-\mathbf{j}+2\mathbf{k}$ Use correct method to calculate their scalar product Show value is zero and planes are perpendicular	**B1** **M1** **A1**	[3]
	(ii)	*EITHER*: Carry out a complete strategy for finding a point on l the line of intersection Obtain such a point, e.g. (0, 7, 5) , (1, 0, 1), (5/4, –7/4, 0) *EITHER*: State two equations for a direction vector $a\mathbf{i}+b\mathbf{j}+c\mathbf{k}$ for l, e.g. $3a+b-c=0$ and $a-b+2c=0$ Solve for one ratio, e.g. $a : b$ Obtain $a : b : c = 1 : -7 : -4$, or equivalent State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$ *OR*1: Obtain a second point on l, e.g. (1, 0, 1) Subtract vectors and obtain a direction vector for l Obtain $-\mathbf{i}+7\mathbf{j}+4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r}=\mathbf{i}+\mathbf{k}+\lambda(-\mathbf{i}+7\mathbf{j}+4\mathbf{k})$ *OR*2: Attempt to find the vector product of the two normal vectors Obtain two correct components of the product Obtain $\mathbf{i}-7\mathbf{j}-4\mathbf{k}$, or equivalent State a correct answer, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$ *OR*1: Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $y = 7 - 7x$ Express the third variable in terms of the second Obtain a correct simplified expression, e.g. $z = 5 - 4x$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r}=7\mathbf{j}+5\mathbf{k}+\lambda(\mathbf{i}-7\mathbf{j}-4\mathbf{k})$ *OR*2: Express one variable in terms of a second variable Obtain a correct simplified expression, e.g. $z=5-4x$ Express the same variable in terms of the third Obtain a correct simplified expression e.g. $z=(7+4y)/7$ Form a vector equation for the line Obtain a correct equation, e.g. $\mathbf{r}=\frac{5}{4}\mathbf{i}-\frac{7}{4}\mathbf{j}+\lambda(-\frac{1}{4}\mathbf{i}+\frac{7}{4}\mathbf{j}+\mathbf{k})$	**M1** **A1** **B1** **M1** **A1** **A1**√ **B1** **M1** **A1** **A1**√ **M1** **A1** **A1** **A1**√ **M1** **A1** **M1** **A1** **M1** **A1**√ **M1** **A1** **M1** **A1** **M1** **A1**√	[6]

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9	**(a)**	EITHER:	Use quadratic formula to solve for w	**M1**	
			Use $i^2 = -1$	**M1**	
			Obtain one of the answers $w = \frac{1}{2i+1}$ and $w = -\frac{5}{2i+1}$	**A1**	
			Multiply numerator and denominator of an answer by $-2i + 1$, or equivalent	**M1**	
			Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	**A1**	
		OR1:	Multiply the equation by $1 - 2i$	**M1**	
			Use $i^2 = -1$	**M1**	
			Obtain $5w^2 + 4w(1-2i) - (1-2i)^2 = 0$, or equivalent	**A1**	
			Use quadratic formula or factorise to solve for w	**M1**	
			Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	**A1**	
		OR2:	Substitute $w = x + iy$ and form equations for real and imaginary parts	**M1**	
			Use $i^2 = -1$	**M1**	
			Obtain $(x^2 - y^2) - 4xy + 4x - 1 = 0$ and $2(x^2 - y^2) + 2xy + 4y + 2 = 0$ o.e.	**A1**	
			Form equation in x only or y only and solve	**M1**	
			Obtain final answers $\frac{1}{5} - \frac{2}{5}i$ and $-1 + 2i$	**A1**	[5]
	(b)	Show a circle with centre $1 + i$		**B1**	
		Show a circle with radius 2		**B1**	
		Show half-line arg $z = \frac{1}{4}\pi$		**B1**	
		Show half-line arg $z = -\frac{1}{4}\pi$		**B1**	
		Shade the correct region		**B1**	[5]

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10 (i)	Separate variables correctly and integrate at least one side	**M1**	
	Integrate and obtain term kt, or equivalent	**A1**	
	Carry out a relevant method to obtain A and B such that $\frac{1}{x(4-x)} \equiv \frac{A}{x} + \frac{B}{4-x}$, or equivalent	**M1***	
	Obtain $A = B = \frac{1}{4}$, or equivalent	**A1**	
	Integrate and obtain terms $\frac{1}{4}\ln x - \frac{1}{4}\ln(4-x)$, or equivalent	**A1**√	
	EITHER: Use a pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
	Obtain correct answer in any form, e.g. $\ln x - \ln(4-x) = 4kt - \ln 9$, or $\ln\left(\frac{x}{4-x}\right) = 4kt - 8k$	**A1**	
	Use a second pair of limits and determine k	**DM1**	
	Obtain the given exact answer correctly	**A1**	
	OR: Use both pairs of limits in a definite integral	**M1***	
	Obtain the given exact answer correctly	**A1**	
	Substitute k and either pair of limits in an expression containing $p\ln x$, $q\ln(4-x)$ and rt and evaluate a constant	**DM1**	
	Obtain $\ln\frac{x}{4-x} = t\ln 3 - \ln 9$ or equivalent	**A1**	[9]
(ii)	Substitute $x = 3.6$ and solve for t	**M1**	
	Obtain answer $t = 4$	**A1**	[2]

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