Cambridge International AS Level

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MATHEMATICS **9709/21**

Paper 2 **October/November 2016**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √ " marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1 (i)	Carry out method for solving quadratic equation in 3^x Obtain at least $3^x = 7$ Use logarithms to solve an equation of the form $3^x = k$ where $k > 0$ Obtain 1.77	**M1** **A1** **M1** **A1**	[4]
(ii)	State ± 1.77, following positive answer from part (i)	**B1**√	[1]
2	State or imply $\ln y = \ln A + px$ Equate gradient of line to p Obtain $p = 0.32$ Substitute to find A Obtain $A = 4.81$ OR 1: $3.17 = \ln A + 5p$ or $4.77 = \ln A + 10p$ Correct attempt to obtain $\ln A$ or p Correct attempt to obtain the other unknown Obtain $A = 4.81$ Obtain $p = 0.32$ OR 2: $e^{3.17} = Ae^{5p}$ or $e^{4.77} = Ae^{10p}$ Correct attempt to obtain p Correct attempt to get A Obtain $A = 4.81$ Obtain $p = 0.32$	**B1** **M1** **A1** **M1** **A1** **B1** **M1** **M1** **A1** **A1** **B1** **M1** **M1** **A1** **A1**	[5]
3	Differentiate to obtain $4\cos 2x + 10\sin 2x$ Equate first derivative to zero and arrange to $\tan 2x = ...$ Obtain $\tan 2x = -0.4$ Carry out correct method for finding at least one value of x, dependent *M Obtain $x = 1.38$ Obtain $x = 2.95$ and no others between 0 and π	**B1** ***M1** **A1** **DM1** **A1** **A1**	[6]
4 (i)	Integrate to obtain $2e^{2x} + 5x$ Apply limits correctly and equate to 100 Rearrange and apply logarithms correctly to reach $a = ...$ Confirm given result $a = \frac{1}{2}\ln(50 + e^{-2a} - 5a)$	**B1** **M1** **M1** **A1**	[4]
(ii)	Use the iterative formula correctly at least once Obtain final answer 1.854 Show sufficient iterations to justify accuracy to 3 dp or show sign change in interval $(1.8535, 1.8545)$	**M1** **A1** **B1**	[3]

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5 (i)	Use $\cos 2x = 2\cos^2 x - 1$ and attempt factorisation of numerator Obtain $(2\cos x + 1)(\cos x + 4)$ Confirm given result $2\cos x + 1$	**M1** **A1** **A1**	[3]
(ii)	Express integrand as $2\cos 2x + 1$ Integrate to obtain $\sin 2x + x$ Apply limits correctly to integral of form $k_1 \sin 2x + k_2 x$ Obtain 2π	**B1** **B1** **M1** **A1**	[4]
6	Differentiate $4xy$ to obtain $4y + 4x\frac{dy}{dx}$ Differentiate y^2 to obtain $2y\frac{dy}{dx}$ Equate attempt of derivative of left-hand side to zero Substitute $(1, 3)$ to find numerical value of derivative Obtain $-\frac{18}{10}$ or $-\frac{9}{5}$ Obtain $\frac{10}{18}$ or $\frac{5}{9}$ as gradient of normal, following their numerical value of derivative Form equation of normal at $(1, 3)$ Obtain $5x - 9y + 22 = 0$ or equivalent of requested form	**B1** **B1** **M1** **M1** **A1** **A1**√ **M1** **A1**	[8]
7 (i)	Substitute $x = -3$, equate to zero and obtain $27a + 3b = 39$ or equivalent Substitute $x = -2$ and equate to 18 Obtain $8a + 2b = 6$ or equivalent Solve a relevant pair of linear equations for a and b Obtain $a = 2$ and $b = -5$	**B1** **M1** **A1** **M1** **A1**	[5]
(ii) (a)	Attempt division by $x + 3$ at least as far as $2x^2 + kx$ Obtain quotient $2x^2 - 3x + 4$ Calculate discriminant of 3-term quadratic expression, or equivalent Obtain -23 and conclude appropriately	**M1** **A1** **M1** **A1**	[4]
(b)	State $\cos y = -\frac{1}{3}$ Obtain 109.5, dependent *B Obtain –109.5 and no others between –180 and 180, dependent *B	***B1** **B1** **DB1**	[3]

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