Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/73**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2015**

1 hour 15 minutes

* 5 2 5 3 4 9 2 5 8 3 *

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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1 It is known that the number, $\boldsymbol{N}$, of words contained in the leading article each day in a certain newspaper can be modelled by a normal distribution with mean 352 and variance 29. A researcher takes a random sample of 10 leading articles and finds the sample mean, $\overline{\boldsymbol{N}}$, of $\boldsymbol{N}$.

(i) State the distribution of $\overline{\boldsymbol{N}}$, giving the values of any parameters. [2]

(ii) Find $\mathrm{P}(\overline{\boldsymbol{N}} > 354)$. [3]

2 The number of calls received per 5-minute period at a large call centre has a Poisson distribution with mean λ, where $\lambda > 30$. If more than 55 calls are received in a 5-minute period, the call centre is overloaded. It has been found that the probability of being overloaded during a randomly chosen 5-minute period is 0.01. Use the normal approximation to the Poisson distribution to obtain a quadratic equation in $\sqrt{\lambda}$ and hence find the value of λ. [5]

3 From a random sample of 65 people in a certain town, the proportion who own a bicycle was noted. From this result an $\alpha\%$ confidence interval for the proportion, $\boldsymbol{p}$, of all people in the town who own a bicycle was calculated to be $0.284 < \boldsymbol{p} < 0.516$.

(i) Find the proportion of people in the sample who own a bicycle. [1]

(ii) Calculate the value of α correct to 2 significant figures. [4]

4 A random variable $\boldsymbol{X}$ has probability density function given by

$$\mathrm{f}(\boldsymbol{x}) = \begin{cases} \boldsymbol{k}(4 - \boldsymbol{x}^2) & -2 \leqslant \boldsymbol{x} \leqslant 2, \\ 0 & \text{otherwise,} \end{cases}$$

where $\boldsymbol{k}$ is a constant.

(i) Show that $\boldsymbol{k} = \frac{3}{32}$. [3]

(ii) Sketch the graph of $\boldsymbol{y} = \mathrm{f}(\boldsymbol{x})$ and hence write down the value of $\mathrm{E}(\boldsymbol{X})$. [2]

(iii) Find $\mathrm{P}(\boldsymbol{X} < 1)$. [3]

5 **(a)** Narika has a die which is known to be biased so that the probability of throwing a 6 on any throw is $\frac{1}{100}$. She uses an approximating distribution to calculate the probability of obtaining no 6s in 450 throws. Find the percentage error in using the approximating distribution for this calculation. [4]

(b) Johan claims that a certain six-sided die is biased so that it shows a 6 less often than it would if the die were fair. In order to test this claim, the die is thrown 25 times and it shows a 6 on only 2 throws. Test at the 10% significance level whether Johan's claim is justified. [5]

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6 Parcels arriving at a certain office have weights $\boldsymbol{W}$ kg, where the random variable $\boldsymbol{W}$ has mean μ and standard deviation 0.2. The value of μ used to be 2.60, but there is a suspicion that this may no longer be true. In order to test at the 5% significance level whether the value of μ has increased, a random sample of 75 parcels is chosen. You may assume that the standard deviation of $\boldsymbol{W}$ is unchanged.

(i) The mean weight of the 75 parcels is found to be 2.64 kg. Carry out the test. [4]

(ii) Later another test of the same hypotheses at the 5% significance level, with another random sample of 75 parcels, is carried out. Given that the value of μ is now 2.68, calculate the probability of a Type II error. [5]

7 The diameter, in cm, of pistons made in a certain factory is denoted by $\boldsymbol{X}$, where $\boldsymbol{X}$ is normally distributed with mean μ and variance σ^2. The diameters of a random sample of 100 pistons were measured, with the following results.

$$\boldsymbol{n} = 100 \qquad \Sigma \boldsymbol{x} = 208.7 \qquad \Sigma \boldsymbol{x}^2 = 435.57$$

(i) Calculate unbiased estimates of μ and σ^2. [3]

The pistons are designed to fit into cylinders. The internal diameter, in cm, of the cylinders is denoted by $\boldsymbol{Y}$, where $\boldsymbol{Y}$ has an independent normal distribution with mean 2.12 and variance 0.000 144. A piston will not fit into a cylinder if $\boldsymbol{Y} - \boldsymbol{X} < 0.01$.

(ii) Using your answers to part **(i)**, find the probability that a randomly chosen piston will not fit into a randomly chosen cylinder. [6]

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