UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **October/November 2011**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

* 8 5 1 1 7 4 2 3 2 9 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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1 The random variable X has the distribution Po(1.3). The random variable Y is defined by $Y = 2X$.

(i) Find the mean and variance of Y. [3]

(ii) Give a reason why the variable Y does not have a Poisson distribution. [1]

2 An engineering test consists of 100 multiple-choice questions. Each question has 5 suggested answers, only one of which is correct. Ashok knows nothing about engineering, but he claims that his general knowledge enables him to get more questions correct than just by guessing. Ashok actually gets 27 answers correct. Use a suitable approximating distribution to test at the 5% significance level whether his claim is justified. [5]

3 Three coats of paint are sprayed onto a surface. The thicknesses, in millimetres, of the three coats have independent distributions $N(0.13, 0.02^2)$, $N(0.14, 0.03^2)$ and $N(0.10, 0.01^2)$. Find the probability that, at a randomly chosen place on the surface, the total thickness of the three coats of paint is less than 0.30 millimetres. [5]

4 The volumes of juice in bottles of Apricola are normally distributed. In a random sample of 8 bottles, the volumes of juice, in millilitres, were found to be as follows.

332 334 330 328 331 332 329 333

(i) Find unbiased estimates of the population mean and variance. [3]

A random sample of 50 bottles of Apricola gave unbiased estimates of 331 millilitres and 4.20 millilitres^2 for the population mean and variance respectively.

(ii) Use this sample of size 50 to calculate a 98% confidence interval for the population mean. [3]

(iii) The manufacturer claims that the mean volume of juice in all bottles is 333 millilitres. State, with a reason, whether your answer to part **(ii)** supports this claim. [1]

5 The management of a factory thinks that the mean time required to complete a particular task is 22 minutes. The times, in minutes, taken by employees to complete this task have a normal distribution with mean μ and standard deviation 3.5. An employee claims that 22 minutes is not long enough for the task. In order to investigate this claim, the times for a random sample of 12 employees are used to test the null hypothesis $\mu = 22$ against the alternative hypothesis $\mu > 22$ at the 5% significance level.

(i) Show that the null hypothesis is rejected in favour of the alternative hypothesis if $\bar{x} > 23.7$ (correct to 3 significant figures), where $\bar{x}$ is the sample mean. [3]

(ii) Find the probability of a Type II error given that the actual mean time is 25.8 minutes. [4]

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6 Customers arrive at an enquiry desk at a constant average rate of 1 every 5 minutes.

(i) State one condition for the number of customers arriving in a given period to be modelled by a Poisson distribution. [1]

Assume now that a Poisson distribution is a suitable model.

(ii) Find the probability that exactly 5 customers will arrive during a randomly chosen 30-minute period. [2]

(iii) Find the probability that fewer than 3 customers will arrive during a randomly chosen 12-minute period. [3]

(iv) Find an estimate of the probability that fewer than 30 customers will arrive during a randomly chosen 2-hour period. [4]

7

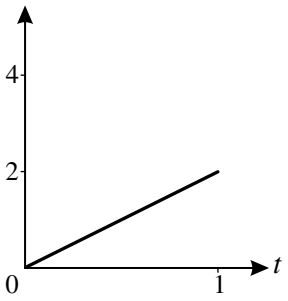


Fig. 1

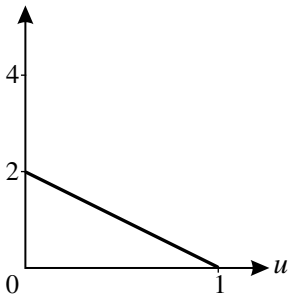


Fig. 2

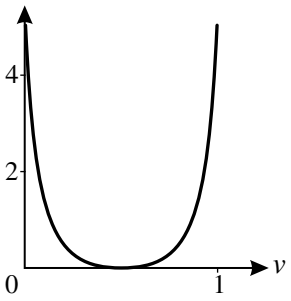


Fig. 3

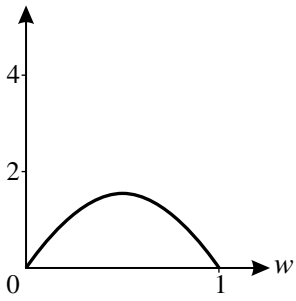


Fig. 4

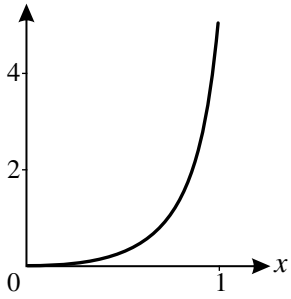


Fig. 5

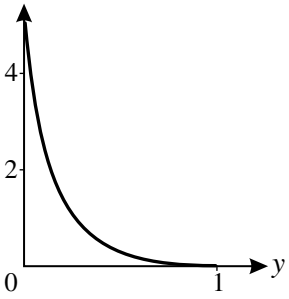


Fig. 6

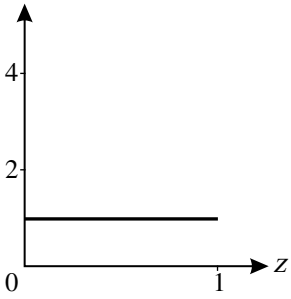


Fig. 7

Each of the random variables T, U, V, W, X, Y and Z takes values between 0 and 1 only. Their probability density functions are shown in Figs 1 to 7 respectively.

(i) **(a)** Which of these variables has the largest median? [1]

(b) Which of these variables has the largest standard deviation? Explain your answer. [2]

(ii) Use Fig. 2 to find $\mathrm{P}(U < 0.5)$. [2]

(iii) The probability density function of X is given by

$$\mathrm{f}(x) = \begin{cases} ax^n & 0 \leqslant x \leqslant 1, \\ 0 & \text{otherwise,} \end{cases}$$

where a and n are positive constants.

(a) Show that $a = n + 1$. [3]

(b) Given that $\mathrm{E}(X) = \frac{5}{6}$, find a and n. [4]

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