UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

* 4 0 8 2 6 2 7 7 4 0 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

JC11 11_9709_33/RP
© UCLES 2011

1 Expand $\frac{16}{(2+x)^2}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 The equation of a curve is $y = \frac{e^{2x}}{1+e^{2x}}$. Show that the gradient of the curve at the point for which $x = \ln 3$ is $\frac{9}{50}$. [4]

3 **(i)** Express $8\cos\theta + 15\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence solve the equation $8\cos\theta + 15\sin\theta = 12$, giving all solutions in the interval $0° < \theta < 360°$. [4]

4 During an experiment, the number of organisms present at time t days is denoted by N, where N is treated as a continuous variable. It is given that

$$\frac{dN}{dt} = 1.2e^{-0.02t}N^{0.5}.$$

When $t = 0$, the number of organisms present is 100.

(i) Find an expression for N in terms of t. [6]

(ii) State what happens to the number of organisms present after a long time. [1]

5 It is given that $\int_1^a x\ln x\,dx = 22$, where a is a constant greater than 1.

(i) Show that $a = \sqrt{\left(\frac{87}{2\ln a - 1}\right)}$. [5]

(ii) Use an iterative formula based on the equation in part **(i)** to find the value of a correct to 2 decimal places. Use an initial value of 6 and give the result of each iteration to 4 decimal places. [3]

6 The complex number w is defined by $w = -1 + i$.

(i) Find the modulus and argument of w^2 and w^3, showing your working. [4]

(ii) The points in an Argand diagram representing w and w^2 are the ends of a diameter of a circle. Find the equation of the circle, giving your answer in the form $|z - (a + bi)| = k$. [4]

© UCLES 2011

7 The polynomial $\mathrm{p}(x)$ is defined by

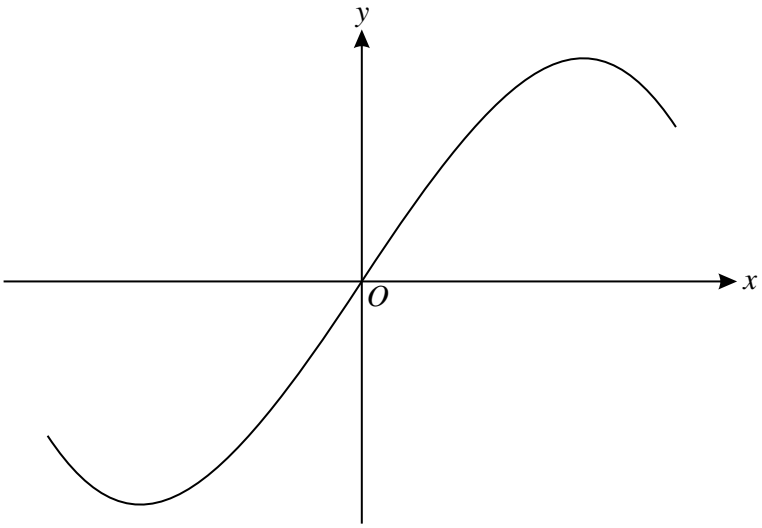
$$\mathrm{p}(x) = ax^3 - x^2 + 4x - a,$$

where a is a constant. It is given that $(2x - 1)$ is a factor of $\mathrm{p}(x)$.

(i) Find the value of a and hence factorise $\mathrm{p}(x)$. [4]

(ii) When a has the value found in part **(i)**, express $\dfrac{8x - 13}{\mathrm{p}(x)}$ in partial fractions. [5]

8



The diagram shows the curve with parametric equations

$$x = \sin t + \cos t, \quad y = \sin^3 t + \cos^3 t,$$

for $\frac{1}{4}\pi < t < \frac{5}{4}\pi$.

(i) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -3 \sin t \cos t$. [3]

(ii) Find the gradient of the curve at the origin. [2]

(iii) Find the values of t for which the gradient of the curve is 1, giving your answers correct to 2 significant figures. [4]

9 The line l has equation $\mathbf{r} = \begin{pmatrix} a \\ 1 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 3 \\ -2 \end{pmatrix}$, where a is a constant. The plane p has equation $2x - 2y + z = 10$.

(i) Given that l does not lie in p, show that l is parallel to p. [2]

(ii) Find the value of a for which l lies in p. [2]

(iii) It is now given that the distance between l and p is 6. Find the possible values of a. [5]

© UCLES 2011

10 **(i)** Use the substitution $u = \tan x$ to show that, for $n \neq -1$,

$$\int_0^{\frac{1}{4}\pi} (\tan^{n+2} x + \tan^n x)\,\mathrm{d}x = \frac{1}{n+1}. \qquad [4]$$

(ii) Hence find the exact value of

(a) $\int_0^{\frac{1}{4}\pi} (\sec^4 x - \sec^2 x)\,\mathrm{d}x,$ [3]

(b) $\int_0^{\frac{1}{4}\pi} (\tan^9 x + 5\tan^7 x + 5\tan^5 x + \tan^3 x)\,\mathrm{d}x.$ [3]

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.