UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **October/November 2010**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

3712617301

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

© UCLES 2010

1 Expand $(1 + 2x)^{-3}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [3]

2 The parametric equations of a curve are

$$x = \frac{t}{2t + 3}, \qquad y = \mathrm{e}^{-2t}.$$

Find the gradient of the curve at the point for which $t = 0$. [5]

3 The complex number w is defined by $w = 2 + \mathrm{i}$.

(i) Showing your working, express w^2 in the form $x + \mathrm{i}y$, where x and y are real. Find the modulus of w^2. [3]

(ii) Shade on an Argand diagram the region whose points represent the complex numbers z which satisfy

$$|z - w^2| \leqslant |w^2|.$$ [3]

4 It is given that $\mathrm{f}(x) = 4\cos^2 3x$.

(i) Find the exact value of $\mathrm{f}'\left(\frac{1}{9}\pi\right)$. [3]

(ii) Find $\int \mathrm{f}(x)\,\mathrm{d}x$. [3]

5 Show that $\displaystyle\int_0^7 \frac{2x + 7}{(2x + 1)(x + 2)}\,\mathrm{d}x = \ln 50$. [7]

6 The straight line l passes through the points with coordinates $(-5, 3, 6)$ and $(5, 8, 1)$. The plane p has equation $2x - y + 4z = 9$.

(i) Find the coordinates of the point of intersection of l and p. [4]

(ii) Find the acute angle between l and p. [4]

7 **(i)** Given that $\displaystyle\int_1^a \frac{\ln x}{x^2}\,\mathrm{d}x = \tfrac{2}{5}$, show that $a = \tfrac{5}{3}(1 + \ln a)$. [5]

(ii) Use an iteration formula based on the equation $a = \tfrac{5}{3}(1 + \ln a)$ to find the value of a correct to 2 decimal places. Use an initial value of 4 and give the result of each iteration to 4 decimal places. [3]

© UCLES 2010

8 **(i)** Express $(\sqrt{6})\cos\theta + (\sqrt{10})\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of α correct to 2 decimal places. [3]

(ii) Hence, in each of the following cases, find the smallest positive angle θ which satisfies the equation

(a) $(\sqrt{6})\cos\theta + (\sqrt{10})\sin\theta = -4$, [2]

(b) $(\sqrt{6})\cos\frac{1}{2}\theta + (\sqrt{10})\sin\frac{1}{2}\theta = 3$. [4]

9 A biologist is investigating the spread of a weed in a particular region. At time t weeks after the start of the investigation, the area covered by the weed is $A\ \text{m}^2$. The biologist claims that the rate of increase of A is proportional to $\sqrt{(2A - 5)}$.

(i) Write down a differential equation representing the biologist's claim. [1]

(ii) At the start of the investigation, the area covered by the weed was $7\ \text{m}^2$ and, 10 weeks later, the area covered was $27\ \text{m}^2$. Assuming that the biologist's claim is correct, find the area covered 20 weeks after the start of the investigation. [9]

10 The polynomial $\text{p}(z)$ is defined by

$$\text{p}(z) = z^3 + mz^2 + 24z + 32,$$

where m is a constant. It is given that $(z + 2)$ is a factor of $\text{p}(z)$.

(i) Find the value of m. [2]

(ii) Hence, showing all your working, find

(a) the three roots of the equation $\text{p}(z) = 0$, [5]

(b) the six roots of the equation $\text{p}(z^2) = 0$. [6]

© UCLES 2010

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

University of Cambridge International Examinations is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of University of Cambridge Local Examinations Syndicate (UCLES), which is itself a department of the University of Cambridge.