Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

0259910944

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages. Any blank pages are indicated.

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1 In a certain country, 20 540 adults out of a population of 6 012 300 have a degree in medicine.

(a) Use an approximating distribution to calculate the probability that, in a random sample of 1000 adults in this country, there will be fewer than 4 adults who have a degree in medicine. [4]

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(b) Justify the approximating distribution used in part **(a)**. [2]

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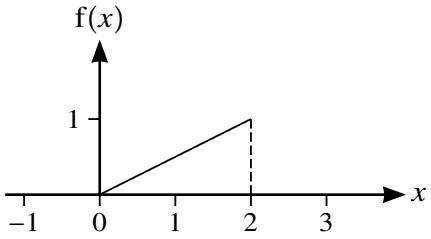
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2 (a)



The graph of the function f is a straight line segment from (0, 0) to (2, 1).

Show that f could be a probability density function. [2]

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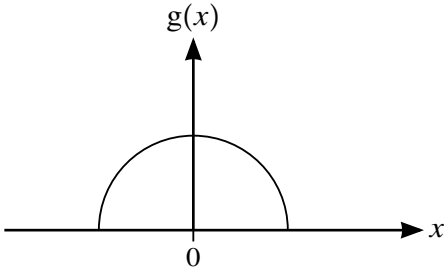
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(b)



The graph of the function g is a semicircle, centre (0, 0), entirely above the x-axis.

Given that g is a probability density function, find the radius of the semicircle. [2]

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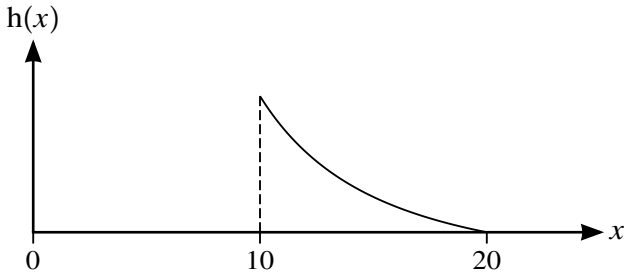
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(c)



The time, X minutes, taken by a large number of students to complete a test has probability density function h, as shown in the diagram.

(i) Without calculation, use the diagram to explain how you can tell that the median time is less than 15 minutes. [1]

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It is now given that

$$\mathrm{h}(x) = \begin{cases} \dfrac{40}{x^2} - \dfrac{1}{10} & 10 \leqslant x \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(ii) Find the mean time. [3]

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3 In the past, the annual amount of wheat produced per farm by a large number of similar sized farms in a certain region had mean 24.0 tonnes and standard deviation 5.2 tonnes. Last summer a new fertiliser was used by all the farms, and it was expected that the mean amount of wheat produced per farm would be greater than 24.0 tonnes. In order to test whether this was true, a scientist recorded the amounts of wheat produced by a random sample of 50 farms last summer. He found that the value of the sample mean was 25.8 tonnes.

Stating a necessary assumption, carry out the test at the 1% significance level. [6]

4 A certain train journey takes place every day throughout the year. The time taken, in minutes, for the journey is normally distributed with variance 11.2.

(a) The mean time for a random sample of n of these journeys was found. A 94% confidence interval for the population mean time was calculated and was found to have a width of 1.4076 minutes, correct to 4 decimal places.

Find the value of n. [3]

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(b) A passenger noted the times for 50 randomly chosen journeys in January, February and March.

Give a reason why this sample is unsuitable for use in finding a confidence interval for the population mean time. [1]

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(c) A researcher took 4 random samples and a 94% confidence interval for the population mean was found from each sample.

Find the probability that exactly 3 of these confidence intervals contain the true value of the population mean. [2]

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5 Large packets of rice are packed in cartons, each containing 20 randomly chosen packets. The masses of these packets are normally distributed with mean 1010 g and standard deviation 3.4 g. The masses of the cartons, when empty, are independently normally distributed with mean 50 g and standard deviation 2.0 g.

(a) Find the variance of the masses of full cartons. [2]

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Small packets of rice are packed in boxes. The total masses of full boxes are normally distributed with mean 6730 g and standard deviation 15.0 g. The masses of the boxes and cartons are distributed independently of each other.

(b) Find the probability that the mass of a randomly chosen full carton is more than three times the mass of a randomly chosen full box. [5]

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6 A sample of 5 randomly selected values of a variable X is as follows:

$$1 \quad 2 \quad 6 \quad 1 \quad a$$

where $a > 0$.

Given that an unbiased estimate of the variance of X calculated from this sample is $\frac{11}{2}$, find the value of a. [3]

7 The number of accidents per week at a certain factory has a Poisson distribution. In the past the mean has been 1.9 accidents per week. Last year, the manager gave all his employees a new booklet on safety. He decides to test, at the 5% significance level, whether the mean number of accidents has been reduced. He notes the number of accidents during 4 randomly chosen weeks this year.

(a) State suitable null and alternative hypotheses for the test. [1]

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(b) Find the critical region for the test and state the probability of a Type I error. [6]

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(c) State what is meant by a Type I error in this context. [1]

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(d) During the 4 randomly chosen weeks there are a total of 3 accidents.

State the conclusion that the manager should reach. Give a reason for your answer. [2]

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(e) Assuming that the mean remains 1.9 accidents per week, use a suitable approximation to calculate the probability that there will be more than 100 accidents during a 52-week period. [4]

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Additional Page

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