Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

0013731051

MATHEMATICS **9709/53**

Paper 5 Probability & Statistics 1 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

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1 Two fair coins are thrown at the same time repeatedly until a pair of heads is obtained. The number of throws taken is denoted by the random variable X.

(a) State the value of $\mathrm{E}(X)$. [1]

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(b) Find the probability that exactly 5 throws are required to obtain a pair of heads. [1]

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(c) Find the probability that fewer than 7 throws are required to obtain a pair of heads. [2]

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2 Anil is a candidate in an election. He received 40% of the votes. A random sample of 120 voters is chosen.

Use an approximation to find the probability that, of the 120 voters, between 36 and 54 inclusive voted for Anil. [5]

3 The random variable X takes the values 1, 2, 3, 4. It is given that $P(X = x) = kx(x + a)$, where k and a are constants.

(a) Given that $P(X = 4) = 3P(X = 2)$, find the value of a and the value of k. [4]

(b) Draw up the probability distribution table for X, giving the probabilities as numerical fractions. [1]

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(c) Given that $E(X) = 3.2$, find $Var(X)$. [2]

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4 The times taken, in minutes, to complete a cycle race by 19 cyclists from each of two clubs, the Cheetahs and the Panthers, are represented in the following back-to-back stem-and-leaf diagram.

Cheetahs						Panthers					
			9	8	7	4					
8	7	3	2	0	8	6	8				
		9	8	7	9	1	7	8	9	9	
6	5	3	3	1	10	2	3	4	4	5	6
		9	8	2	11	1	2	8			
				4	12	0	6				

Key: 7 $\vert$9$\vert$ 1 means 97 minutes for Cheetahs and 91 minutes for Panthers

(a) Find the median and the interquartile range of the times of the Cheetahs. [3]

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The median and interquartile range for the Panthers are 103 minutes and 14 minutes respectively.

(b) Make two comparisons between the times taken by the Cheetahs and the times taken by the Panthers. [2]

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Another cyclist, Kenny, from the Cheetahs also took part in the race. The mean time taken by the 20 cyclists from the Cheetahs was 99 minutes.

(c) Find the time taken by Kenny to complete the race. [3]

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5 Jasmine throws two ordinary fair 6-sided dice at the same time and notes the numbers on the uppermost faces. The events A and B are defined as follows.

A: The sum of the two numbers is less than 6.

B: The difference between the two numbers is at most 2.

(a) Determine whether or not the events A and B are independent. [4]

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(b) Find $\mathrm{P}(B \mid A')$. [3]

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6 The mass of grapes sold per day by a large shop can be modelled by a normal distribution with mean 28 kg. On 10% of days less than 16 kg of grapes are sold.

(a) Find the standard deviation of the mass of grapes sold per day. [3]

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The mass of grapes sold on any day is independent of the mass sold on any other day.

(b) 12 days are chosen at random.

Find the probability that less than 16 kg of grapes are sold on more than 2 of these 12 days. [3]

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(c) In a random sample of 365 days, on how many days would you expect the mass of grapes sold to be within 1.3 standard deviations of the mean? [4]

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7 **(a)** Find the number of different arrangements of the 10 letters in the word CASABLANCA in which the two Cs are **not** together. [3]

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(b) Find the number of different arrangements of the 10 letters in the word CASABLANCA which have an A at the beginning, an A at the end and exactly 3 letters between the 2 Cs. [3]

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Five letters are selected from the 10 letters in the word CASABLANCA.

(c) Find the number of different selections in which the five letters include at least two As and at most one C. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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