Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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7943719452

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics 3 **May/June 2023**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

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1 Solve the inequality $|5x - 3| < 2|3x - 7|$. [4]

2 Solve the equation $\ln(2x^2 - 3) = 2\ln x - \ln 2$, giving your answer in an exact form. [3]

3 **(a)** On an Argand diagram, sketch the locus of points representing complex numbers z satisfying $|z + 3 - 2i| = 2$. [2]

(b) Find the least value of $|z|$ for points on this locus, giving your answer in an exact form. [2]

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4 Solve the equation $2\cos x - \cos \frac{1}{2}x = 1$ for $0 \leqslant x \leqslant 2\pi$. [5]

5 The complex number $2 + yi$ is denoted by a, where y is a real number and $y < 0$. It is given that $f(a) = a^3 - a^2 - 2a$.

(a) Find a simplified expression for $f(a)$ in terms of y. [3]

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(b) Given that $\mathrm{Re}\big(f(a)\big) = -20$, find $\arg a$. [3]

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6 The equation $\cot \frac{1}{2}x = 3x$ has one root in the interval $0 < x < \pi$, denoted by α.

(a) Show by calculation that α lies between 0.5 and 1. [2]

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(b) Show that, if a sequence of positive values given by the iterative formula

$$x_{n+1} = \frac{1}{3}\left(x_n + 4\tan^{-1}\left(\frac{1}{3x_n}\right)\right)$$

converges, then it converges to α. [2]

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(c) Use this iterative formula to calculate α correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

7 The equation of a curve is $3x^2 + 4xy + 3y^2 = 5$.

(a) Show that $\frac{dy}{dx} = -\frac{3x + 2y}{2x + 3y}$. [4]

(b) Hence find the exact coordinates of the two points on the curve at which the tangent is parallel to $y + 2x = 0$. [5]

8 **(a)** The variables x and y satisfy the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4 + 9y^2}{\mathrm{e}^{2x+1}}.$$

It is given that $y = 0$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x. [7]

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(b) State what happens to the value of y as x tends to infinity. Give your answer in an exact form. [1]

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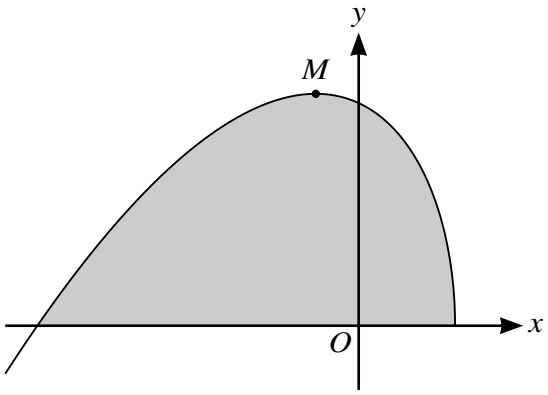
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9 Let $\mathrm{f}(x) = \dfrac{2x^2 + 17x - 17}{(1 + 2x)(2 - x)^2}$.

(a) Express $\mathrm{f}(x)$ in partial fractions. [5]

(b) Hence show that $\int_0^1 \mathrm{f}(x)\,\mathrm{d}x = \frac{5}{2} - \ln 72$. [5]

10



The diagram shows the curve $y = (x + 5)\sqrt{3 - 2x}$ and its maximum point M.

(a) Find the exact coordinates of M. [5]

(b) Using the substitution $u = 3 - 2x$, find by integration the area of the shaded region bounded by the curve and the x-axis. Give your answer in the form $a\sqrt{13}$, where a is a rational number. [5]

11 The points A and B have position vectors $\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$ and $2\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively. The line l has equation $\mathbf{r} = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$.

(a) Show that l does not intersect the line passing through A and B. [5]

(b) Find the position vector of the foot of the perpendicular from A to l. [4]

Additional Page

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