Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

8595284852

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **May/June 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

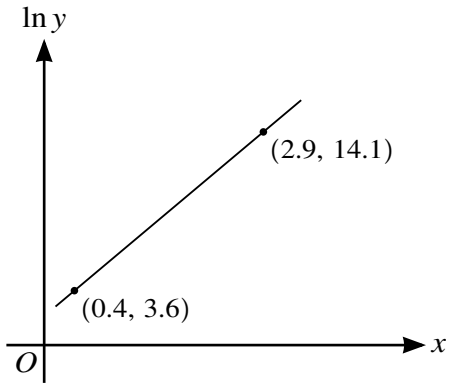
© UCLES 2023

1 Solve the equation

$$\sec^2\theta + 5\tan^2\theta = 9 + 17\sec\theta$$

for $0° < \theta < 360°$. [5]

2



The variables x and y satisfy the equation $y = Ae^{(A-B)x}$, where A and B are constants. The graph of $\ln y$ against x is a straight line passing through the points (0.4, 3.6) and (2.9, 14.1), as shown in the diagram.

Find the values of A and B correct to 3 significant figures. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

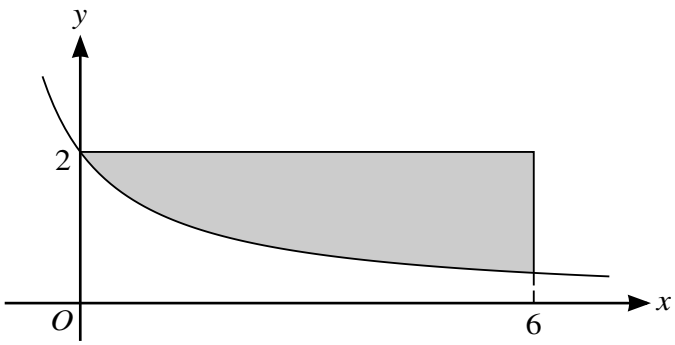
.....

.....

.....

.....

3



The diagram shows part of the curve $y = \dfrac{6}{2x + 3}$. The shaded region is bounded by the curve and the lines $x = 6$ and $y = 2$.

Find the exact area of the shaded region, giving your answer in the form $a - \ln b$, where a and b are integers. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

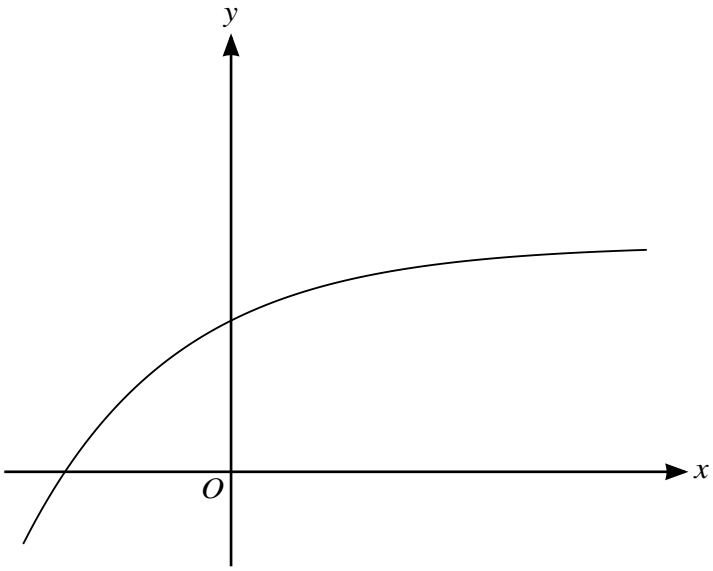
.....

.....

.....

.....

4 (a)



The diagram shows the graph of $y = 3 - e^{-\frac{1}{2}x}$.

On the diagram, sketch the graph of $y = |5x - 4|$, and show that the equation $3 - e^{-\frac{1}{2}x} = |5x - 4|$ has exactly two real roots. [2]

It is given that the two roots of $3 - e^{-\frac{1}{2}x} = |5x - 4|$ are denoted by α and β, where $\alpha < \beta$.

(b) Show by calculation that α lies between 0.36 and 0.37. [2]

.....

.....

.....

.....

.....

(c) Use the iterative formula $x_{n+1} = \frac{1}{5}\left(7 - e^{-\frac{1}{2}x_n}\right)$ to find β correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

.....

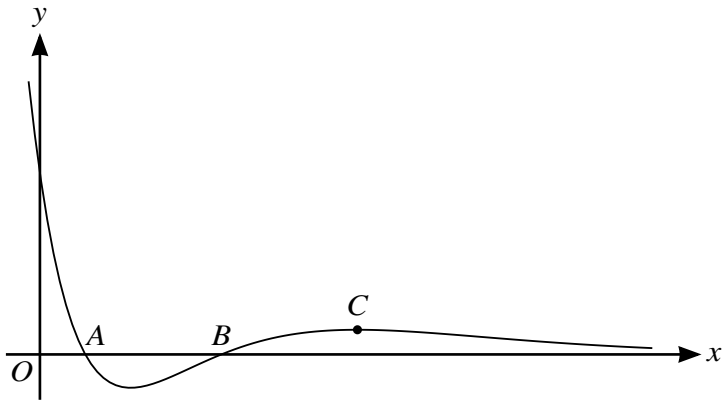
.....

.....

.....

.....

5



The diagram shows the curve with equation $y = e^{-\frac{1}{2}x}(x^2 - 5x + 4)$. The curve crosses the x-axis at the points A and B, and has a maximum at the point C.

(a) Find the exact gradient of the curve at B. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the exact coordinates of C. [4]

6 **(a)** Show that $4\sin\left(\theta+\frac{1}{3}\pi\right)\cos\left(\theta-\frac{1}{3}\pi\right) \equiv \sqrt{3}+2\sin 2\theta$. [4]

(b) Find the exact value of $4\sin\frac{17}{24}\pi\cos\frac{1}{24}\pi$. [2]

(c) Find the exact value of $\int_0^{\frac{1}{8}\pi} 4\sin\left(2x+\frac{1}{3}\pi\right)\cos\left(2x-\frac{1}{3}\pi\right)\mathrm{d}x$. [4]

7 A curve has parametric equations

$$x = \frac{2t + 3}{t + 2}, \qquad y = t^2 + at + 1,$$

where a is a constant. It is given that, at the point P on the curve, the gradient is 1.

(a) Show that the value of t at P satisfies the equation

$$2t^3 + (a + 8)t^2 + (4a + 8)t + 4a - 1 = 0. \qquad [4]$$

(b) It is given that $(t + 1)$ is a factor of

$$2t^3 + (a + 8)t^2 + (4a + 8)t + 4a - 1.$$

Find the value of a. [2]

.....

.....

.....

.....

.....

.....

.....

.....

(c) Hence show that P is the only point on the curve at which the gradient is 1. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

© UCLES 2023