Cambridge Assessment International Education

Cambridge International A Level

MATHEMATICS **9709/61**

Paper 6 Probability & Statistics 2 **May/June 2023**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	20540/6012300 = 0.0034163	**B1**	
	[1000 × 0.0034163 = 3.4163]		
	Po(3.4163)	**B1**	Could be implied by expression seen.
	$e^{-their\ '3.4163'}(1 + 3.4163 + \frac{3.4163^2}{2!} + \frac{3.4163^3}{3!})$ OR $e^{-their\ '3.4163'}$(1 + 3.4163 + 5.8356+6.6453) or 0.03283 + 0.1122 +0.1916 + 0.21819)	**M1**	Allow any λ. Allow with one end error. Must see expression.
	= 0.555 (3sf)	**A1**	CAO **SC** No working: B1 B1 (Po must be stated) B1 correct answer (max 3/4). **SC** Binomial: B1 B0 B1 correct answer (max 2/4).
		4	
1(b)	$n = 1000 > 50$	**B1**	Must show comparison with 50.
	$np = 3.4163 < 5$	**B1**	Must show comparison with 5.
		2	**SC B1**: $n > 50$ (or n large), $np < 5$. **SC B1**: n large, p small.

Question	Answer	Marks	Guidance
2(a)	$\frac{1}{2} \times 2 \times 1$ or $\int_0^2 \frac{1}{2}x\,dx = 1$, which is the correct area under a pdf.	**B1**	Calculation and result.
	$f(x) \geqslant 0$	**B1**	Condone $f(x) > 0$ or 'Line is above x-axis' OE.
		2	

Question	Answer	Marks	Guidance
2(b)	$\frac{1}{2}\pi r^2 = 1$	**M1**	Area of semi-circle equated to 1 OE. Missing factor of ½ gets M1A0.
	$r = \sqrt{\frac{2}{\pi}}$ or 0.798 (3sf)	**A1**	
		2	
2(c)(i)	Area to the left of 15 is greater than 0.5	**B1**	OE, e.g. 'The distribution of X is skewed to the right / positively skewed, suggesting the median will be less than the mid-point of the interval.' or 'The distribution of X is skewed to the right / positively skewed' or 'It is a decreasing function suggesting the median will be less than the mid-point of the interval'.
		1	
2(c)(ii)	$\int_{10}^{20} (\frac{40}{x} - \frac{x}{10})\mathrm{d}x$	**M1**	Integration of xh(x) attempted. Ignore limits.
	$\left[40\ln x - \frac{x^2}{20}\right]_{10}^{20}$	**A1**	Correct integration and limits (can be implied by final answer).
	= 40ln 2 – 15 or 12.7 (3sf)	**A1**	
		3	

Question	Answer	Marks	Guidance
3	Assume SD still = 5.2	**B1**	OE i.e. 'Assume the SD remains unchanged'.
	H_0: $\mu = 24.0$ H_1: $\mu > 24.0$	**B1**	Or population mean; not just mean.
	$\frac{25.8-24.0}{\frac{5.2}{\sqrt{50}}}$	**M1**	For standardising (could be implied). Must have $\sqrt{50}$.
	= 2.448	**A1**	Or P($\overline{X} > 25.8$) = 0.0071 .
	'2.448' > 2.326	**M1**	Or 0.0071 < 0.01 . For valid comparison.
	[Reject H_0] There is evidence that (mean) amount of wheat is greater.	**A1FT**	OE. FT their z_{calc}. In context, not definite, eg not 'Mean amount of wheat is greater' No contradictions CV method: CV= 25.71 M1A1 25.71<25.8 M1 A1FT or CV=24.09 M1 A1 24.09>24 M1 A1FT.
		6	

Question	Answer	Marks	Guidance
4(a)	$z \times \sqrt{\frac{11.2}{n}} = 1.4076 \div 2$	**M1**	Any z, but must be a z.
	z = 1.881 or 1.882	**B1**	
	$[n = \left(\frac{1.881}{0.7038}\right)^2 \times 11.2]$ n = 80	**A1**	Must be a whole number.
		3	

Question	Answer	Marks	Guidance
4(b)	Jan, Feb and March not typical of whole year.	**B1**	Or, e.g., weather is different at different times of year.
		1	
4(c)	$0.94^3 \times 0.06 \times 4$	**M1**	
	= 0.199 (3 sf)	**A1**	
		2	

Question	Answer	Marks	Guidance
5(a)	$2.0^2 + 20 \times 3.4^2$	**M1**	
	= 235.2	**A1**	
		2	
5(b)	E(C – 3B) = 50 + 20×1010 – 3×6730 or 60	**B1**	
	Var(C – 3B) = '235.2' + 9×15^2 or 2260.2	**M1**	FT *their* values from **(a)**.
	[C – 3B ~ N('60', '2260.2')] = $\frac{0-60}{\sqrt{2260.2}}$ [= –1.262]	**M1**	Standardising with their values (could be implied).
	1 – Φ('–1.262') = Φ('1.262')	**M1**	Probability area consistent with their values.
	= 0.897 (3 sf)	**A1**	
		5	

Question	Answer	Marks	Guidance
6	$\frac{5}{4}\left(\frac{1+2^2+6^2+1+a^2}{5}-\left(\frac{1+2+6+1+a}{5}\right)^2\right)=\frac{11}{2}$ or $\frac{1}{4}\left((42+a^2)-\frac{(10+a^2)}{5}\right)=\frac{11}{2}$	**M1***	OE attempted or e.g., $\frac{42+a^2}{5}-\left(\frac{10+a}{5}\right)^2=\frac{22}{5}$. Allow use of biased i.e., without $\frac{5}{4}$.
	$4a^2 - 20a + 0 = 0$ or $a^2 - 5a + 0 = 0$	**DM1**	Two- or three-term quadratic equation in a, with at least two terms correct.
	$a = 5$	**A1**	Ignore $a = 0$, if seen.
		3	

Question	Answer	Marks	Guidance
7(a)	H_0: $\lambda = 7.6$ [or 1.9] H_1: $\lambda < 7.6$ [or 1.9]	**B1**	Or Population mean = 7.6 or μ (not just 'mean'). Or Population mean < 7.6 or μ.
		1	

Question	Answer	Marks	Guidance
7(b)	Mean = 7.6	**B1**	Seen.
	$P(X \leqslant 2) = e^{-7.6}(1 + 7.6 + \frac{7.6^2}{2})$ [= 0.0188 or 0.0187]	**M1**	OE.
	$P(X \leqslant 3) = e^{-7.6}(1 + 7.6 + \frac{7.6^2}{2} + \frac{7.6^3}{3!})$ [= 0.0554 or 0.0553]	**M1**	OE. Expression must be seen in at least one probability calculation.
	0.0188 or 0.0187 and 0.0554 or 0.0553	**A1**	A1 for both values.
	Critical region is $X \leqslant 2$	**A1**	Dep on both M marks. **SC** No Poisson expression seen in either prob scores B1 for 0.0188 or 0.0187 and B1 for 0.0554 or 0.0553 and B1 for CR.
	P(Type I error) = $P(X \leqslant 2)$ = 0.0188 or 0.0187 (3 sf)	**B1FT**	FT *their* $P(X \leqslant 2)$ or *their* CR.
		6	
7(c)	Concluding that the (mean) no. of accidents has reduced when it has not.	**B1**	OE. Must be in context. Accept: 'It is believed that the booklet has helped to improve safety when actually it has not'.
		1	
7(d)	3 not in critical region.	**M1**	FT their CR or $P(X \leq 3) = 0.0554 > 0.05$.
	No evidence mean number of accidents has decreased.	**A1FT**	In context. Cannot be a definite statement, e.g., 'mean number accidents has not decreased'.
		2	

Question	Answer	Marks	Guidance
7(e)	N(98.8, 98.8)	**B1**	May be implied.
	$\frac{100.5 - 98.8}{\sqrt{98.8}}$ [= 0.171]	**M1**	For standardising (could be implied by correct answer). Allow with wrong or no continuity correction.
	1 – Φ(‘0.171’)	**M1**	For probability area consistent with their working.
	= 0.432 (3 sf)	**A1**	
		4	

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