Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2023**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

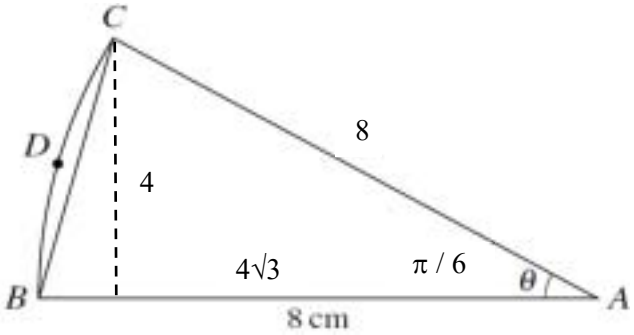
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Question	Answer	Marks	Guidance
1	$4\sin\theta + \tan\theta = 0 \Rightarrow 4\sin\theta + \frac{\sin\theta}{\cos\theta}[= 0]$	**M1**	For use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$. BOD if θ missing.
	$\Rightarrow \sin\theta(4\cos\theta + 1)[= 0 \Rightarrow \sin\theta = 0 \text{ or}] \cos\theta = -\frac{1}{4}$	**M1**	WWW Factorise, not divide by $\sin\theta$ or $\tan\theta$. May see $\tan\theta(4\cos\theta + 1)[= 0]$ or $\sin\theta(4 + \sec\theta)[= 0]$.
	$\theta = 104.5°$	**A1**	AWRT 1.82 rads A0. Ignore answers outside (0, 180°). If M1 M0, **SC B1** for $\theta = 104.5°$ max 2/3.
		3	

Question	Answer	Marks	Guidance
2(a)	$16 + 96x + 216x^2$	**B2, 1, 0**	ISW (higher powers of x). Terms may be in any order or presented as a list.
		2	
2(b)	$1 - 10x + 40x^2$	**B2, 1, 0**	ISW (higher powers of x). Terms may be in any order or presented as a list.
		2	
2(c)	$(16 \times 40) - (10 \times 96) + (1 \times 216)$	**M1**	*Their* 3 products which would give the term in x^2 (FT *their* values). Look for $640 - 960 + 216$.
	-104	**A1**	Condone $-104x^2$.
		2	

Question	Answer	Marks	Guidance
3	{Stretch} {factor 2} {in y-direction}	**B2, 1, 0**	**B2** for fully correct, **B1** with two elements correct. {} indicates different elements.
	{Translation} $\begin{pmatrix}\{-6\}\\\{0\}\end{pmatrix}$	**B2, 1, 0**	**B2** for fully correct, **B1** with two elements correct. {} indicates different elements.
		4	Transformations may be in either order.

Question	Answer	Marks	Guidance
4	$\frac{1}{2}\times 8^2\times\theta=\frac{16\pi}{3}\Rightarrow\theta=\frac{\pi}{6}$	**B1**	SOI OE e.g. $\frac{2\pi}{12}$, $0.524(3\text{s.f.})$ Use of degrees acceptable throughout provided conversion used in formulae for sector area and arc length.
	Arc length $=8\times their\frac{\pi}{6}$ [= 4.1887…]	**M1**	OE FT *their* θ. Look for $\frac{4\pi}{3}$.
	$[BC=]\ 2\times 8\sin\left(\frac{1}{2}\times their\frac{\pi}{6}\right)$ [= 4.1411…]	**M1**	Attempt to find BC or BC^2 (see alt. methods below) FT *their* θ. Look for $16\sin\frac{\pi}{12}$ or $4\sqrt{6}-4\sqrt{2}$.
	Perimeter = 8.33	**A1**	AWRT Must be combined into one term.

Question	Answer	Marks	Guidance
4	**Alternative methods for Question 4: 2nd M1 mark (use normal scheme for the other marks)**		
	ALT 1 $BC^2 = 8^2 + 8^2 - 2\times 8\times 8\cos\left(their\,\frac{\pi}{6}\right)\left[\Rightarrow BC = 4.14\ldots\right]$ **ALT 2** $BC^2 = \left(8-4\sqrt{3}\right)^2 + 4^2\left[\Rightarrow BC = 4.14\ldots\right]$  **ALT 3** $\frac{BC}{\sin\left(\frac{\pi}{6}\right)} = \frac{8}{\sin\left(\frac{5\pi}{12}\right)}\left[\Rightarrow BC = 4.14\ldots\right]$		ALT 1 Substitute into correct cosine rule. FT *their* θ Look for $128 - 64\sqrt{3}$ ALT 2 Find lengths 4 and $4\sqrt{3}$ then use Pythagoras in the left hand triangle. ALT 3 Substitute into correct sine rule.
		4	

Question	Answer	Marks	Guidance
5	$kx-k=-\frac{1}{2x} \Rightarrow 2kx^2-2kx+1[=0]$ OR quadratic in y: $x=\frac{y+k}{k} \Rightarrow y=-\frac{1}{2\left(\frac{y+k}{k}\right)} \Rightarrow 2y^2+2ky+k=0$	***M1**	OE e.g. $kx^2-kx+\frac{1}{2}[=0]$, $x^2-x+\frac{1}{2k}[=0]$ Equate line and curve to form 3-term quadratic (all terms on one side).
	$b^2-4ac[=0] \Rightarrow \left([-]2k\right)^2-4(2k)(1)[=0]$ or $4k^2-8k\ [=0 \Rightarrow 4k(k-2)=0]$ OR using equation in y: $(2k)^2-4(2)(k)=0$	**DM1**	Use discriminant correctly with their a,b,c not in quadratic formula. DM0 if x still present. May see $k^2-4(k)\left(\frac{1}{2}\right)=0$ or $1-4\left(\frac{1}{2k}\right)=0$.
	$k=2$ only	**A1**	If DM0 then $k=2$, award A0 XP then B0 B0 Allow A1 even if divides by k to solve. If $k=0$ also present but uses $k=2$, award A1.
	$4x^2-4x+1=0\left[\Rightarrow (2x-1)^2=0\right] \Rightarrow x=\frac{1}{2}$	**B1**	
	$y=2\times\frac{1}{2}-2=-1$	**B1**	

Question	Answer	Marks	Guidance
5	**Alternative method for Q5**		
	$\frac{dy}{dx}=\frac{1}{2x^2}$ or $\frac{1}{2}x^{-2}$	***M1**	Differentiate $-\frac{1}{2x}$ M0 for $2x^{-2}$. No errors.
	$\left[y=\right]\frac{1}{2x^2}x-\frac{1}{2x^2}=-\frac{1}{2x}$ or $\frac{1}{x}=\frac{1}{2x^2}$ $\left[\Rightarrow 2x^2-x=0\right]$	**DM1**	Sub *their* $\frac{dy}{dx}$ into equation of line or set gradient = k to form equation in x.
	$x=\frac{1}{2}$ only	**A1**	If DM0 then $x=\frac{1}{2}$, award A0XP then B0 B0.
	$y=\left[2\times\frac{1}{2}-2\right]=-1$	**B1**	
	$k=2$	**B1**	
		5	

Question	Answer	Marks	Guidance
6(a)	$2(2p-6)=p+\frac{p^2}{6} \Rightarrow \frac{p^2}{6}-3p+12[=0]$ OR $(2p-6)-\frac{p^2}{6}=p-(2p-6) \Rightarrow \frac{p^2}{6}-3p+12[=0]$ OR $\frac{1}{6}d^2+d[=0]$	***M1**	Correct method leading to formation of a 3-term quadratic in p (all terms on one side) or 2-term quadratic in d. OE e.g. $p^2-18p+72[=0]$, $\frac{1}{2}p^2-9p+36[=0]$.
	$p^2-18p+72[=0] \Rightarrow (p-6)(p-12)[=0]$ **or** $\frac{18\pm\sqrt{(-18)^2-4(1)(72)}}{2}$ OR $d\left(\frac{1}{6}d+1\right)[=0] \Rightarrow d=-6$	**DM1**	Solve a 3-term quadratic in p by factorisation, formula or completing the square or solve a 2-term quadratic in d by factorisation.
	$p = 12$ only	**A1**	Since $p = 6$ gives $d = 0$. If *M1 DM0 then $p = 12$ only, award **SC B1**, max 2/3 marks. A0 XP if error in either factor and $p=12$ only. $p=12$ only by trial and improvement 3/3.
		3	

Question	Answer	Marks	Guidance
6(b)	For GP $r = \left[\dfrac{\frac{2p-6}{p^2}}{6}\right] = \dfrac{18}{24}\left[=\dfrac{3}{4}\right]$	**B1**	OE SOI.
	Sum to infinity $= \dfrac{24}{1-\frac{3}{4}} = 96$	**B1 FT**	FT *their* value of p if used correctly to find r (B0 if 'p' used) provided $\lvert r\rvert < 1$. e.g. $p = 18 \Rightarrow \left[S_\infty =\right] \dfrac{54}{1-\frac{5}{9}} = 121.5$.
		2	

Question	Answer	Marks	Guidance
7(a)	[Greatest =] 5	**B1**	No inequality required.
	[Least =] –1	**B1**	No inequality required.
			Condone $(-1,5)$ or equivalent.
		2	

Question	Answer	Marks	Guidance
7(b)		**B1**	One complete cycle starting and finishing at $y = 2$. Maximum and minimum in correct quadrants. Shape and curvature approximately correct.
		B1 FT	Maximum and minimum (indicated on y-axis with numbers or lines, or labelled on graph). FT *their* greatest and least values. Award **B1** for 5 and -1 even if *their* values were incorrect in **(a)**.
		2	
7(c)	1	**B1**	WWW
		1	

Question	Answer	Marks	Guidance
8(a)	$1+\frac{2a}{7-a}=\frac{5}{2}\left[\Rightarrow \frac{2a}{7-a}=\frac{3}{2} \Rightarrow 7a=21\right]\Rightarrow a=\ldots$ OR $1+\frac{2a}{7-a}=\frac{5}{2}\left[\Rightarrow (7-a)+2a=\frac{5}{2}(7-a)\ [\Rightarrow 7a=21]\right]\Rightarrow a=\ldots$	**M1**	OE Substitute $x=7$ then solve for a via legitimate mathematical steps. Condone sign errors only.
	$a=3$	**A1**	If M0, **SC B1** for $a=3$ with no working.
	$\mathrm{f}(5)=1+\frac{2(their3)}{5-their3}=4\left[\Rightarrow 4b-2=4\right]\Rightarrow b=\ldots$ OR $\mathrm{gf}(5)=b\left(1+\frac{2(their3)}{5-their3}\right)-2\ \left[\Rightarrow 4b-2=4\right]\Rightarrow b=\ldots$	**M1**	Evaluate f(5)**,** either separately or within gf then solve for b via legitimate mathematical steps. Condone sign errors only. FT *their* a value.
	$b=\frac{3}{2}$	**A1**	OE e.g. $\frac{6}{4}$, 1.5 .
		4	
8(b)	$x>1$	**B1**	Accept $(1,\infty)$ or {*: * > 1} where * is any variable. B0 for $\mathrm{f}^{-1}(x)>1$ or $\mathrm{f}(x)>1$ or $y>1$.
		1	

Question	Answer	Marks	Guidance
8(c)	EITHER $x-1=\dfrac{6}{y-3}$ $\left[\Rightarrow (y-3)(x-1)=6\right]$ OR $x=1+\dfrac{6}{y-3} \Rightarrow x(y-3)=(y-3)+6$	***M1**	OE $y-1=\dfrac{6}{x-3} \Rightarrow (x-3)(y-1)=6$. OE $y=1+\dfrac{6}{x-3} \Rightarrow y(x-3)=(x-3)+6$. Allow *M1 for use of *their* 3 from **(a)**.
	$y-3=\dfrac{6}{x-1}$ or $y(x-1)=3x+3$	**DM1**	OE $x-3=\dfrac{6}{y-1}$ or $x(y-1)=3y+3$. Allow DM1 for use of *their* 3 from **(a)**.
	$\left[\mathrm{f}^{-1}(x)\right]=3+\dfrac{6}{x-1}$	**A1**	OE Correct answer e.g. $\dfrac{3x+3}{x-1}$ ISW. Must be in terms of x.
			*M1 DM1 possible for 'a' used, but A0 so max 2/3.
		3	

Question	Answer	Marks	Guidance
9(a)	$\frac{\mathrm{d}V}{\mathrm{d}h}=\frac{4}{3}\times 3(25+h)^2$ [= 4900 when h = 10]	**B1**	Correct expression for $\frac{\mathrm{d}V}{\mathrm{d}h}$.
	$\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}t} \Rightarrow \textit{their}\ "4(25+10)^2"\times\frac{\mathrm{d}h}{\mathrm{d}t}=500\Rightarrow\frac{\mathrm{d}h}{\mathrm{d}t}=\left[\frac{500}{4900}\right]$	**M1**	Use chain rule correctly to find a numerical expression for $\frac{\mathrm{d}h}{\mathrm{d}t}$. Accept e.g. $\frac{500}{2500+2000+400}$.
	$\frac{\mathrm{d}h}{\mathrm{d}t}=0.102\left[\mathrm{cms}^{-1}\right]$	**A1**	AWRT OE e.g. $\frac{5}{49}$ ISW.
		3	
9(b)	$\frac{\mathrm{d}V}{\mathrm{d}t}=\frac{\mathrm{d}V}{\mathrm{d}h}\times\frac{\mathrm{d}h}{\mathrm{d}t}\Rightarrow 500=\ \textit{their}\ "4(25+h)^2"\times 0.075$	***M1**	SOI Use chain rule correctly to form equation in h.
	$\left[(25+h)^2=\frac{5000}{3}\right]\Rightarrow h=[15.8\,248\ldots]$	**DM1**	Solve quadratic to find h. Exact value of h is $\sqrt{\frac{5000}{3}}-25$ or $\frac{50\sqrt{6}}{3}-25$ $h+25=40.82\ldots$
	$V=69900\ \mathrm{cm}^3$	**A1**	AWRT ISW Look for 698(88.5) .
		3	

Question	Answer	Marks	Guidance
10(a)	$[\pi]\int \frac{16}{(2x-1)^4}[\mathrm{d}x]=[\pi]\int 16(2x-1)^{-4}[\mathrm{d}x]=[\pi]\left(-\frac{16}{3\times 2\times(2x-1)^3}\right)$	***M1**	Integrate y^2 (power incr. by 1 or div by *their* new power). M0 if more than 1 error or $-\frac{16}{6}x(2x-1)^{-3}$.
	$[\pi]\left(-\frac{16}{3\times 2\times(2x-1)^3}\right)$	**A1**	OE e.g. $\left(-\frac{8}{3}(2x-1)^{-3}\right)$.
	$[\pi]\left(-\frac{16}{6\times 8}+\frac{16}{6\times 1}\right)\ \left[=[\pi]\frac{112}{48}=[\pi]\frac{7}{3}\right]$	**DM1**	Sub correct limits into *their* integral: F$\left(\frac{3}{2}\right)$ − F(1). Must see at least $\left(-\frac{1}{3}+\frac{8}{3}\right)$. Allow 1 sign error. Decimal: 2.33 π or 7.33 .
	Volume of cylinder $\left[=\pi\times 1^2\times\frac{1}{2}\right]=\frac{1}{2}\pi$ OR $[\pi]\int_1^{1.5}1[\mathrm{d}x]=\frac{1}{2}\pi$	**B1**	$\frac{1}{2}\pi$ or $\pm\pi\left(\frac{3}{2}-1\right)$ seen.
	Volume of revolution $\left[=\frac{7}{3}\pi-\frac{1}{2}\pi\right]=\frac{11}{6}\pi$	**A1**	A0 for 5.76 (not exact). If DM0 for insufficient substitution, or B0, **SC B1** for $\frac{11}{6}\pi$.
		5	

Question	Answer	Marks	Guidance
10(b)	$\left[\frac{\mathrm{d}y}{\mathrm{d}x}=\right]\left\{-8(2x-1)^{-3}\right\}\{\times 2\}$	**B2, 1, 0**	OE B1 for each correct element in {}.
	At B gradient $= -2$	**B1**	
	Eqn of tangent $y-1=$ *their* $"-2"\left(x-\frac{3}{2}\right)$ OR Eqn of normal $y-1=$ *their* $"\frac{1}{2}"\left(x-\frac{3}{2}\right)$	**M1**	SOI Following differentiation OE e.g. $y=-2x+4$ or $y=\frac{1}{2}x+\frac{1}{4}$. (Must have $m_N=-\frac{1}{m_T}$ for M1).
	Tangent crosses x-axis at 2 **or** normal crosses x-axis at $-\frac{1}{2}$	**A1**	SOI For at least one intercept correct or correct integration.
	Area $=\frac{5}{4}$	**A1**	From intercepts: $\frac{1}{2}\times\frac{5}{2}\times 1=\frac{5}{4}$ or $1+\frac{1}{4}=\frac{5}{4}$, from lengths: $\frac{1}{2}\times\sqrt{5}\times\frac{\sqrt{5}}{2}=\frac{5}{4}$ or by integration.
		6	

Question	Answer	Marks	Guidance
11(a)	$6a^2-30a+6a=0$ $[\Rightarrow 6a(a-4)=0]$	**B1**	Sub $x=a$ into $\frac{\mathrm{d}y}{\mathrm{d}x}=0$. May see $a^2-5a+a=0$.
	$a=4$ only	**B1**	
		2	

Question	Answer	Marks	Guidance
11(b)	$\frac{d^2y}{dx^2} = 12x - 30$ or correct values of $\frac{dy}{dx}$ either side of $x = 4$	**M1**	Differentiate $\frac{dy}{dx}$ (mult. by power or dec. power by 1) M0 if no values of $\frac{dy}{dx}$, only signs.
	At $x = 4, \frac{d^2y}{dx^2} > 0 \therefore$ minimum or $\frac{d^2y}{dx^2} = 18 \therefore$ minimum or concludes minimum from $\frac{dy}{dx}$ values	**A1**	WWW A0 XP if $a = 4$ obtained incorrectly in (a) Must see 'minimum'. If M0, **SC B1** for 'minimum' from $\frac{dy}{dx}$ sign diagram.
		2	
11(c)	$[y =] \ \frac{6}{3}x^3 - \frac{30}{2}x^2 + 6(their\ a)x[+c]$	**B1 FT**	Expect $2x^3 - 15x^2 + 24x[+c]$. B1 poss. even if uses 'a' – no value in (a) – max 1/3.
	$-15 = 2(their\ "4")^3 - 15(their\ "4")^2 + 6(their\ "4") + c$	**M1**	Sub $x =$ *their*"4", $y = -15$ into integral (must incl $+c$) Look for $-15 = 128 - 240 + 96 + c$ [$\Rightarrow c = 1$].
	$y = 2x^3 - 15x^2 + 24x + 1$	**A1**	Coefficients must be correct and simplified. Need to see '$y =$' or '$f(x) =$' in the working.
		3	
11(d)	$\frac{dy}{dx} = 6x^2 - 30x + 6(their\ "4")[= 0]$ If correct, $[6](x-1)(x-4)[= 0]$ or $\frac{30 \pm \sqrt{(-30)^2 - 4(6)(24)}}{12}$	**M1**	OE Forming a 3-term quadratic using the given $\frac{dy}{dx}$ and solving by factorisation, formula or completing the square. Check for working in **(b)**.
	Coordinates $(1,12)$	**A1**	Allow $x = 1, y = 12$ (ignore $x = 4$ if present). If M0, award **SC B1** for $(1,12)$.
		2	

Question	Answer	Marks	Guidance
12(a)	$x^2+(y-2)^2=100$	**B1**	OE e.g. $(x-0)^2+(y-2)^2=10^2$ ISW.
		1	
12(b)	Gradient of radius $=\left[\frac{10-2}{6-0}=\right]\frac{4}{3}$ or gradient of tangent $=\frac{-3}{4}$	**M1**	OE SOI Use coordinates to find gradient of radius or differentiate to find m_T e.g. $2x+2(y-2)\frac{dy}{dx}=0\Rightarrow\frac{dy}{dx}=\frac{-3}{4}$ at (6, 10) $y=2+\sqrt{100-x^2}\Rightarrow\frac{dy}{dx}=\frac{1}{2}(100-x^2)^{-\frac{1}{2}}(-2x)=-\frac{3}{4}$.
	Equation of tangent is $y-10=-\frac{3}{4}(x-6)\quad\left[\Rightarrow y=-\frac{3}{4}x+\frac{29}{2}\right]$	**A1**	OE ISW Allow e.g. $\frac{58}{4}$.
		2	
12(c)	Coordinates of centre of circle Q are $\left(0, their\frac{29}{2}\right)$	**M1**	SOI From a linear equation in **(b)**.
	Equation of circle Q is $x^2+\left(y-their\frac{29}{2}\right)^2=\left(\frac{5\sqrt{5}}{2}\right)^2\left[=\frac{125}{4}\right]$	**A1FT**	OE e.g. $(x-0)^2+(y-14.5)^2=31.25$ ISW.
	$x^2+(11-2)^2=100\Rightarrow\ x^2=19$ and $x^2+\left(11-\frac{29}{2}\right)^2=\frac{125}{4}\ \Rightarrow\ x^2=19$ OR e.g. $\frac{125}{4}-\left(y-\frac{29}{2}\right)^2+(y-2)^2=100\Rightarrow 25y=275\Rightarrow y=11$	**B1**	OE e.g. $x=[\pm]\sqrt{19},\ x^2-19=x^2-19$ Correct argument to verify both y-coords are 11 ISW.
		3	

Question	Answer	Marks	Guidance
12(d)	$x^2+\left(-\frac{3}{4}x+\frac{29}{2}-\frac{29}{2}\right)^2=\frac{125}{4}\ \left[\Rightarrow\frac{25}{16}x^2=\frac{125}{4}\Rightarrow x^2=20\right]$ or $y^2-29y+199[=0]$	**M1**	Substitute equation of *their* tangent into equation of *their* circle. May see $y=\sqrt{31.25-x^2}+14.5$.
	$x=\pm2\sqrt{5}$ or $y=\frac{29\mp3\sqrt{5}}{2}$	**A1**	OE e.g. $x=\pm\sqrt{20}$ For 2 x-values or 2 y-values or correct (x,y) pair.
	$y\left[=\left(-\frac{3}{4}\times\pm\sqrt{20}\right)+\frac{29}{2}\right]=\frac{29\mp3\sqrt{5}}{2}$	**A1**	OE e.g. $\frac{58}{4}+\frac{3\sqrt{20}}{4}$, $\frac{58}{4}-\frac{3\sqrt{20}}{4}$ Correct (x,y) pairs.
		3	

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