Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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3849072491

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2022**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 The number of characters in emails sent by a particular company is modelled by the distribution $N(1250, 480^2)$.

Find the probability that the mean number of characters in a random sample of 100 emails sent by the company is more than 1300. [3]

2 Anton believes that 10% of students at his college are left-handed. Aliya believes that this is an underestimate. She plans to carry out a hypothesis test of the null hypothesis $p = 0.1$ against the alternative hypothesis $p > 0.1$, where p is the actual proportion of students at the college that are left-handed. She chooses a random sample of 20 students from the college. She will reject the null hypothesis if at least 5 of these students are left-handed.

(a) Explain what is meant by a Type I error in this context. [1]

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(b) Find the probability of a Type I error in the test. [3]

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(c) Given that the true value of p is 0.3, find the probability of a Type II error in the test. [2]

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3 Batteries of type A are known to have a mean life of 150 hours. It is required to test whether a new type of battery, type B, has a shorter mean life than type A batteries.

(a) Give a reason for using a sample rather than the whole population in carrying out this test. [1]

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A random sample of 120 type B batteries are tested and it is found that their mean life is 147 hours, and an unbiased estimate of the population variance is 225 hours^2.

(b) Test, at the 2% significance level, whether type B batteries have a shorter mean life than type A batteries. [5]

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(c) Calculate a 94% confidence interval for the population mean life of type B batteries. [3]

4 Each box of Seeds & Raisins contains S grams of seeds and R grams of raisins. The weight of a box, when empty, is B grams. S, R and B are independent random variables, where $S \sim N(300, 45)$, $R \sim N(200, 25)$ and $B \sim N(15, 4)$. A full box of Seeds & Raisins is chosen at random.

(a) Find the probability that the total weight of the box and its contents is more than 500 grams. [5]

(b) Find the probability that the weight of seeds in the box is less than 1.4 times the weight of raisins in the box. [5]

5 The number of clients who arrive at an information desk has a Poisson distribution with mean 2.2 per 5-minute period.

(a) Find the probability that, in a randomly chosen 15-minute period, exactly 6 clients arrive at the desk. [3]

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(b) If more than 4 clients arrive during a 5-minute period, they cannot all be served.

Find the probability that, during a randomly chosen 5-minute period, not all the clients who arrive at the desk can be served. [2]

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(c) Use a suitable approximating distribution to find the probability that, during a randomly chosen 1-hour period, fewer than 20 clients arrive at the desk. [4]

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6 A random sample of 5 values of a variable X is given below.

$$2 \quad 3 \quad 3 \quad 5 \quad a$$

(a) Find an expression, in terms of a, for the mean of these values. [1]

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It is given that an unbiased estimate of the population variance of X, using these values, is 4. It is also given that a is positive.

(b) Find and simplify a quadratic equation in terms of a and hence find the value of a. [3]

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7 The random variables X and W have probability density functions f and g defined as follows:

$$\mathrm{f}(x) = \begin{cases} p(a^2 - x^2) & 0 \leqslant x \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

$$\mathrm{g}(w) = \begin{cases} q(a^2 - w^2) & -a \leqslant w \leqslant a, \\ 0 & \text{otherwise,} \end{cases}$$

where a, p and q are constants.

(a) **(i)** Write down the value of $\mathrm{P}(X \geqslant 0)$. [1]

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(ii) Write down the value of $\mathrm{P}(W \geqslant 0)$. [1]

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(iii) Write down an expression for q in terms of p only. [1]

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(b) Given that $E(X) = 3$, find the value of a. [6]

Additional Page

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