Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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3281101778

MATHEMATICS **9709/23**

Paper 2 Pure Mathematics 2 **May/June 2022**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 Given that $y = \dfrac{\ln x}{x^2}$, find the exact value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \mathrm{e}$. [3]

2 **(a)** Sketch, on the same diagram, the graphs of $y = |2x - 9|$ and $y = 5x - 3$. [2]

(b) Solve the equation $|2x - 9| = 5x - 3$. [2]

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3 A curve has equation $e^{2x} \cos 2y + \sin y = 1$.

Find the exact gradient of the curve at the point $(0, \frac{1}{6}\pi)$. [5]

4 **(a)** Use the trapezium rule with three intervals to show that the value of $\int_1^4 \ln x \, \mathrm{d}x$ is approximately $\ln 12$. [4]

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(b) Use a graph of $y = \ln x$ to show that $\ln 12$ is an under-estimate of the true value of $\int_1^4 \ln x \, \mathrm{d}x$. [2]

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5 The polynomial $p(x)$ is defined by

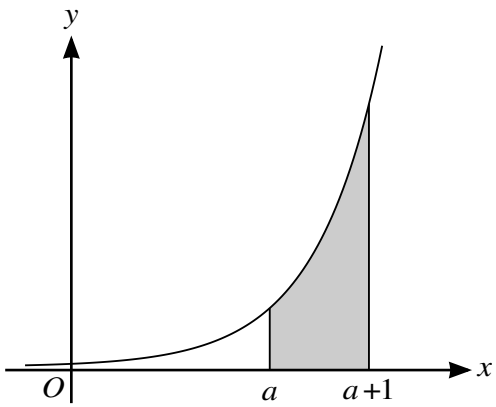
$$p(x) = 2x^3 + ax^2 - 3x - 4,$$

where a is a constant. It is given that $(x - 4)$ is a factor of $p(x)$.

(a) Find the value of a and hence factorise $p(x)$. [4]

(b) Show that the equation $p(e^{3y}) = 0$ has only one real root and find its exact value. [3]

6



The diagram shows the curve $y = 3e^{2x-1}$. The shaded region is bounded by the curve and the lines $x = a$, $x = a + 1$ and $y = 0$, where a is a constant. It is given that the area of the shaded region is 120 square units.

(a) Show that $a = \frac{1}{2}\ln(80 + e^{2a-1}) - \frac{1}{2}$. [5]

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(b) Use an iterative formula, based on the equation in part **(a)**, to find the value of a correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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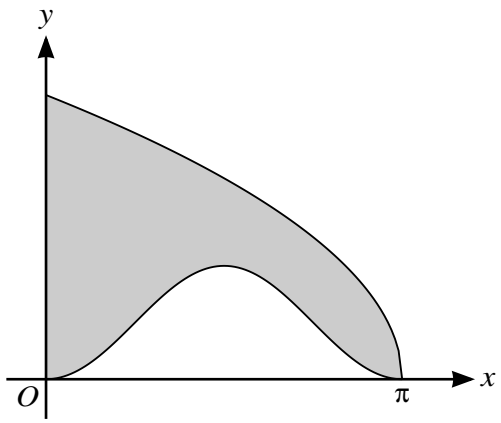
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The diagram shows the curves $y = \sqrt{2\pi - 2x}$ and $y = \sin^2 x$ for $0 \leqslant x \leqslant \pi$. The shaded region is bounded by the two curves and the line $x = 0$.

Find the exact area of the shaded region. [8]

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8 **(a)** Express $3\sin 2\theta \sec\theta + 10\cos(\theta - 30°)$ in the form $R\sin(\theta + \alpha)$ where $R > 0$ and $0° < \alpha < 90°$. Give the value of α correct to 2 decimal places. [6]

(b) Hence solve the equation $3\sin 4\beta \sec 2\beta + 10\cos(2\beta - 30°) = 2$ for $0° < \beta < 90°$. [3]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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