Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

8494117371

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **May/June 2022**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages.

© UCLES 2022

1 **(a)** Express $x^2 - 8x + 11$ in the form $(x + p)^2 + q$ where p and q are constants. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Hence find the exact solutions of the equation $x^2 - 8x + 11 = 1$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2 The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is −15.

Find the sum of the first 50 terms of the progression. [5]

3 The coefficient of x^4 in the expansion of $\left(2x^2 + \frac{k^2}{x}\right)^5$ is a. The coefficient of x^2 in the expansion of $(2kx - 1)^4$ is b.

(a) Find a and b in terms of the constant k. [3]

(b) Given that $a + b = 216$, find the possible values of k. [3]

4 **(a)** Prove the identity $\dfrac{\sin^3\theta}{\sin\theta - 1} - \dfrac{\sin^2\theta}{1 + \sin\theta} \equiv -\tan^2\theta(1 + \sin^2\theta)$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Hence solve the equation

$$\frac{\sin^3\theta}{\sin\theta - 1} - \frac{\sin^2\theta}{1 + \sin\theta} = \tan^2\theta(1 - \sin^2\theta)$$

for $0 < \theta < 2\pi$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

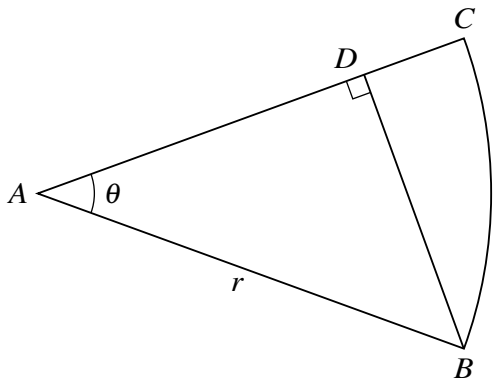
.....

.....

.....

.....

5



The diagram shows a sector ABC of a circle with centre A and radius r. The line BD is perpendicular to AC. Angle CAB is θ radians.

(a) Given that $\theta = \frac{1}{6}\pi$, find the exact area of BCD in terms of r. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

(b) Given instead that the length of BD is $\frac{\sqrt{3}}{2}r$, find the exact perimeter of BCD in terms of r. [4]

6 The function f is defined as follows:

$$\mathrm{f}(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for $\mathrm{f}^{-1}(x)$. [3]

(b) Show that $1 - \frac{8}{x^2 + 4}$ can be expressed as $\frac{x^2 - 4}{x^2 + 4}$ and hence state the range of f. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Explain why the composite function ff cannot be formed. [1]

.....

.....

.....

.....

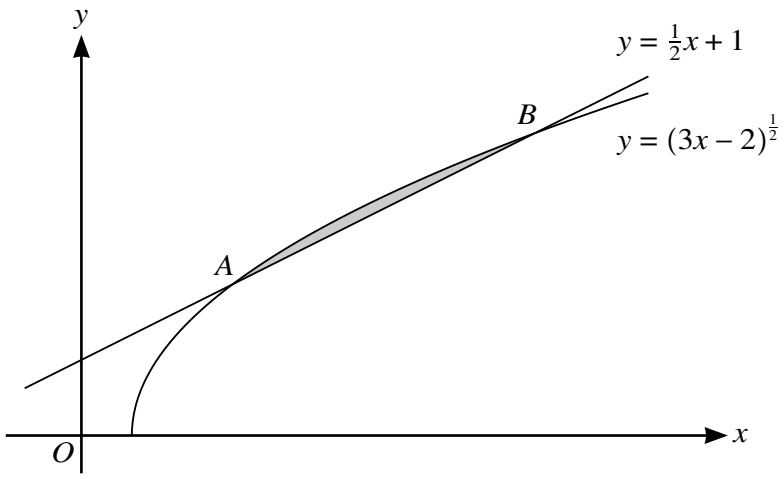
.....

.....

.....

.....

7



The diagram shows the curve with equation $y = (3x - 2)^{\frac{1}{2}}$ and the line $y = \frac{1}{2}x + 1$. The curve and the line intersect at points A and B.

(a) Find the coordinates of A and B. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

(b) Hence find the area of the region enclosed between the curve and the line. [5]

8 **(a)** The curve $y = \sin x$ is transformed to the curve $y = 4\sin\left(\frac{1}{2}x - 30°\right)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the exact solutions of the equation $4\sin\left(\frac{1}{2}x - 30°\right) = 2\sqrt{2}$ for $0° \leqslant x \leqslant 360°$. [3]

9 The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

(a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

(b) Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersects the circle at two distinct points. [6]

10 The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $\left(-1, \frac{9}{2}\right)$.

(a) Determine the nature of the stationary point at $\left(-1, \frac{9}{2}\right)$. [1]

.....

.....

.....

.....

(b) Find the equation of the curve. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

(c) Show that the curve has no other stationary points. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) A point A is moving along the curve and the y-coordinate of A is increasing at a rate of 5 units per second.

Find the rate of increase of the x-coordinate of A at the point where $x = 1$. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2022

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

© UCLES 2022