Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

5436857677

MATHEMATICS **9709/63**

Paper 6 Probability & Statistics 2 **May/June 2021**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 The number of goals scored by a team in a match is independent of other matches, and is denoted by the random variable X, which has a Poisson distribution with mean 1.36. A supporter offers to make a donation of \$5 to the team for each goal that they score in the next 10 matches.

Find the expectation and standard deviation of the amount that the supporter will pay. [5]

2 In the past, the time, in hours, for a particular train journey has had mean 1.40 and standard deviation 0.12. Following the introduction of some new signals, it is required to test whether the mean journey time has decreased.

(a) State what is meant by a Type II error in this context. [1]

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(b) The mean time for a random sample of 50 journeys is found to be 1.36 hours.

Assuming that the standard deviation of journey times is still 0.12 hours, test at the 2.5% significance level whether the population mean journey time has decreased. [5]

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(c) State, with a reason, which of the errors, Type I or Type II, might have been made in the test in part **(b)**. [2]

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3 The local council claims that the average number of accidents per year on a particular road is 0.8. Jane claims that the true average is greater than 0.8. She looks at the records for a random sample of 3 recent years and finds that the total number of accidents during those 3 years was 5.

(a) Assume that the number of accidents per year follows a Poisson distribution.

(i) State null and alternative hypotheses for a test of Jane's claim. [1]

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(ii) Test at the 5% significance level whether Jane's claim is justified. [4]

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(b) Jane finds that the number of accidents per year has been gradually increasing over recent years.

State how this might affect the validity of the test carried out in part **(a)(ii)**. [1]

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4 The masses, m kilograms, of flour in a random sample of 90 sacks of flour are summarised as follows.

$$n = 90 \qquad \Sigma m = 4509 \qquad \Sigma m^2 = 225\,950$$

(a) Find unbiased estimates of the population mean and variance. [3]

(b) Calculate a 98% confidence interval for the population mean. [3]

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(c) Explain why it was necessary to use the Central Limit theorem in answering part **(b)**. [1]

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(d) Find the probability that the confidence interval found in part **(b)** is wholly above the true value of the population mean. [2]

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5 Most plants of a certain type have three leaves. However, it is known that, on average, 1 in 10 000 of these plants have four leaves, and plants with four leaves are called 'lucky'. The number of lucky plants in a random sample of 25 000 plants is denoted by X.

(a) State, with a justification, an approximating distribution for X, giving the values of any parameters. [2]

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Use your approximating distribution to answer parts **(b)** and **(c)**.

(b) Find $P(X \leqslant 3)$. [2]

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(c) Given that $P(X = k) = 2P(X = k + 1)$, find k. [2]

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The number of lucky plants in a random sample of n plants, where n is large, is denoted by Y.

(d) Given that $P(Y \geqslant 1) = 0.963$, correct to 3 significant figures, use a suitable approximating distribution to find the value of n. [3]

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6 Alethia models the length of time, in minutes, by which her train is late on any day by the random variable X with probability density function given by

$$f(x) = \begin{cases} \frac{3}{8000}(x-20)^2 & 0 \leqslant x \leqslant 20, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the probability that the train is more than 10 minutes late on each of two randomly chosen days. [4]

(b) Find E(X). [4]

(c) The median of X is denoted by m.

Show that m satisfies the equation $(m - 20)^3 = -4000$, and hence find m correct to 3 significant figures. [4]

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(d) State one way in which Alethia's model may be unrealistic. [1]

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Additional Page

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