Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME	
CENTRE NUMBER	
CANDIDATE NUMBER	

2355526103

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **May/June 2021**

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

© UCLES 2021

1 **(a)** Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2021

2 **(a)** The graph of $y = \mathrm{f}(x)$ is transformed to the graph of $y = 2\mathrm{f}(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) The curve $y = \sin 2x - 5x$ is reflected in the y-axis and then stretched by scale factor $\frac{1}{3}$ in the x-direction.

Write down the equation of the transformed curve. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

3 The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A\,(2, k)$ $B\,(2.9, 2.8025)$ $C\,(2.99, 2.9800)$ $D\,(2.999, 2.9980)$ $E\,(3, 3)$

(a) Find k, giving your answer correct to 4 decimal places. [1]

.....

.....

.....

.....

(b) Find the gradient of AE, giving your answer correct to 4 decimal places. [1]

.....

.....

.....

.....

.....

.....

The gradients of BE, CE and DE, rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

(c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E. [2]

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2021

4 The coefficient of x in the expansion of $\left(4x + \frac{10}{x}\right)^3$ is p. The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{k}{x^2}\right)^5$ is q.

Given that $p = 6q$, find the possible values of k. [5]

5 The function f is defined by $f(x) = 2x^2 + 3$ for $x \geqslant 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

.....

.....

.....

.....

.....

.....

(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

6 Points A and B have coordinates $(8, 3)$ and (p, q) respectively. The equation of the perpendicular bisector of AB is $y = -2x + 4$.

Find the values of p and q. [4]

7 The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A. A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8 The first, second and third terms of an arithmetic progression are a, $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a, 18 and $b + 3$ respectively.

(a) Find the values of a and b. [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

.....

.....

.....

.....

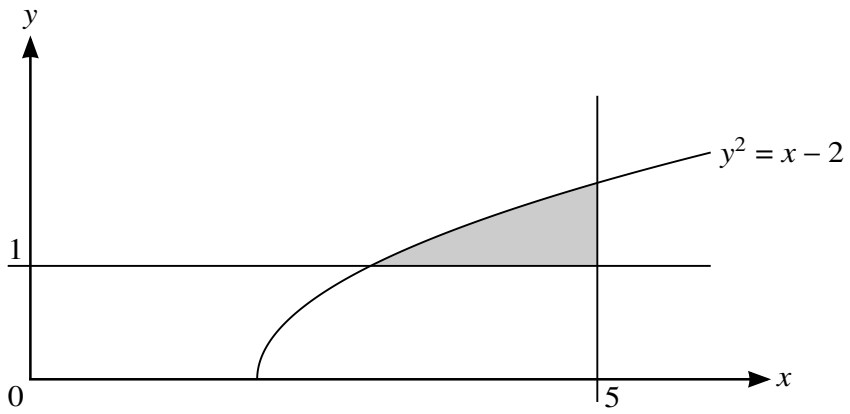
.....

.....

.....

.....

9



The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x-axis.

Find the volume obtained. [6]

10 **(a)** Prove the identity $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} \equiv \frac{4\tan x}{\cos x}$. [4]

(b) Hence solve the equation $\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} = 8\tan x$ for $0 \leqslant x \leqslant \frac{1}{2}\pi$. [3]

11 The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k. [2]

.....

.....

.....

.....

.....

(b) Find the equation of the curve. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find $\frac{d^2y}{dx^2}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) Determine the nature of the stationary point at $(2, -3.5)$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

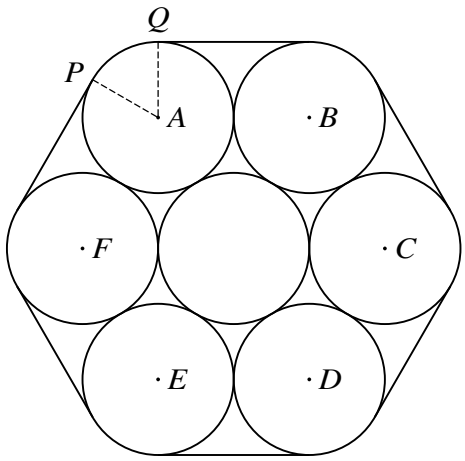
.....

.....

.....

.....

12



The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A, B, C, D, E and F. Points P and Q are situated where straight sections of the rope meet the pipe with centre A.

(a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

.....

.....

.....

.....

.....

.....

(b) Find the length of the rope. [4]

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2021

(c) Find the area of the hexagon $ABCDEF$, giving your answer in terms of $\sqrt{3}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(d) Find the area of the complete region enclosed by the rope. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2021

BLANK PAGE

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.