Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/51**

Paper 5 Probability & Statistics 1 **May/June 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

© UCLES 2021

[Turn over

PUBLISHED

Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

© UCLES 2021

Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

© UCLES 2021

PUBLISHED

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

© UCLES 2021

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

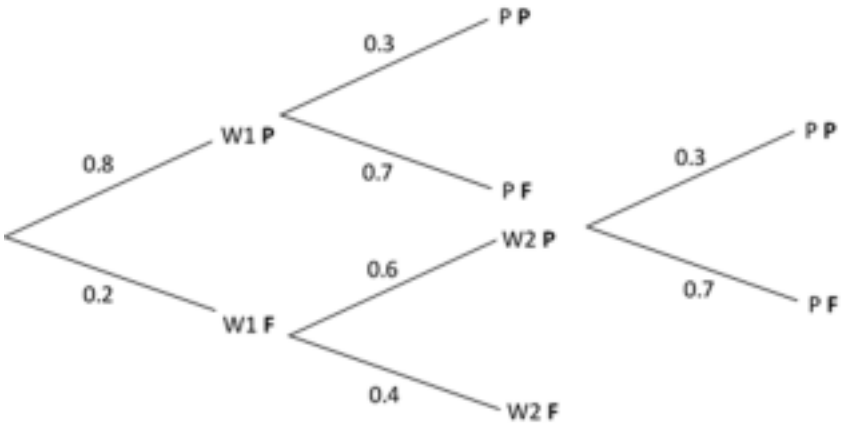
© UCLES 2021

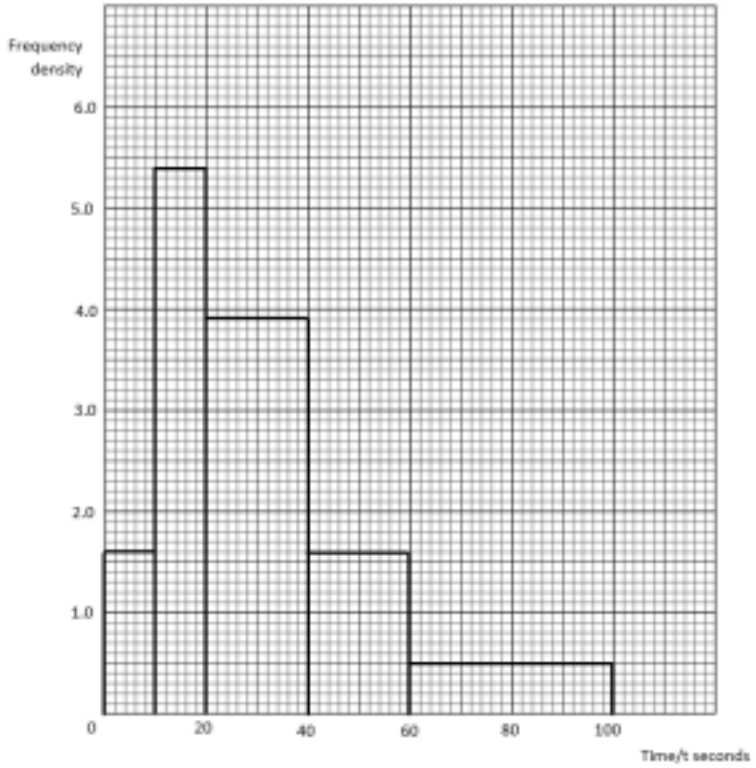
Question	Answer	Marks	Guidance
1	RRRRB ${}^8C_4 \times {}^4C_1 = 280$ BBBBR ${}^8C_1 \times {}^4C_4 = 8$ RRRRR ${}^8C_5 = 56$	M1	${}^8C_x \times {}^4C_y$ with $x + y = 5$. x, y both integers, $1 \leqslant x \leqslant 5$, $0 \leqslant y \leqslant 4$ condone ${}^8C_1 \times 1$
		A1	Two correct outcomes evaluated
		M1	Add 2 or 3 identified correct scenarios only (no additional terms, not probabilities)
	[Total =] 344	A1	WWW, only dependent on 2nd M mark
		4	**SC** not all (or no) scenarios identified **B1** 280 + 8 + 56 **DB1** 344

Question	Answer	Marks	Guidance
2	$\left[P\left(\left(\frac{25.2-(25.5+0.50)}{0.4}\right)<z<\left(\frac{25.2-(25.2-0.50)}{0.4}\right)\right)\right]$ $= P\left(-\frac{0.5}{0.4}<z<\frac{0.5}{0.4}\right)$	M1	Use of $\pm$ Standardisation formula once; no continuity correction, σ^2, $\sqrt{\sigma}$
	$\left[=2\Phi(1.25)-1\right]$ $=2\times0.8944-1$	A1	For AWRT 0.8944 SOI
		M1	Appropriate area $2\Phi - 1$ OE, from final process, must be probability
	0.7888	A1	Accept AWRT 0.789
	Number of rods = 0.7888×500 = 394 or 395	B1FT	Correct or FT *their* 4SF (or better) probability, final answer must be positive integer, not 394.0 or 395.0, no approximation/rounding stated, only 1 answer
		5	

Question	Answer	Marks	Guidance
3(a)	$\left[\frac{8!}{3!}\right] = 6720$	B1	NFWW, must be evaluated
		1	
3(b)	_ _ _ L E D _ _ : With LED together: $\frac{6!}{2!}$	M1	$\frac{6!}{k}$ or $\frac{5!x6}{k}$ $k \geqslant 1$ and no other terms
		M1	$\frac{m}{2!}$, m an integer, $m \geqslant 5$
	360	A1	CAO
		3	
3(c)	Method using _ _ _ A _ D _ _ : Arrange the 6 letters RELESE = $\frac{6!}{3!}$ [= 120]	*M1	$\frac{6!}{3!} \times k$ seen, k an integer > 0
	Multiply by number of ways of placing AD in non-adjacent places = *their* 120 × $^7P_2$ [= 5040]	*M1	$m \times n(n-1)$ or $m \times {}^nC_2$ or $m \times {}^nP_2$, n = 6, 7 or 8, m an integer > 0
	[Probability =] $\frac{\textit{their}\,5040}{\textit{their}\,6720}$	DM1	Denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using 'Total arrangements – Arrangements with A and D together': *Their* 6720 – $\frac{7! \times 2}{3!}$ [= 5040]	*M1	*Their* 6720 – k, k a positive integer
		*M1	$(m-)\frac{7! \times k}{3!}, k = 1, 2$

Question	Answer	Marks	Guidance
	[Probability =] $\frac{their\,5040}{their\,6720}$	DM1	With denominator = *their* **(a)** or correct, dependent on at least one M mark already gained.
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
	Alternative method for Question 3(c)		
	Method using '1 – Probability of arrangements with A and D together': $\frac{7!\times 2}{3!}$ [= 1680]	*M1	$\frac{7!\times k}{3!}, k = 1, 2$
	[Probability =] $\frac{their\,1680}{their\,6720}$	*M1	With denominator = *their* **(a)** or correct
	$1 - \frac{their\,1680}{their\,6720}$	DM1	$1 - m$, $0 < m < 1$, dependent on at least one M mark already gained
	$\frac{5040}{6720}$ or $\frac{3}{4}$ or 0.75	A1	
		4	

Question	Answer	Marks	Guidance
4(a)		B1	Fully correct labelled tree diagram for each pair of branches clearly identifying written and practical, pass and fail for each intersection (no additional branches)
		B1	'One written test' branch all probabilities (or %) correct
		B1	'Two written tests' branch all probabilities (or %) correct, condone additional branches after W2F with probabilities 1 for PF and 0 for PP
		3	
4(b)	[P(W1P) × P(PP) + P(W1F) × P(W2P) × P(PP)] $0.8\times0.3+0.2\times0.6\times0.3$	M1	Consistent with *their* tree diagram or correct
	0.276 or $\frac{69}{250}$	A1	
		2	
4(c)	$P(W1\mid P)=\frac{P(W1\cap\text{Practical})}{P(\text{getting place})}=\frac{0.8\times0.3}{their(b)}\left[=\frac{0.24}{0.276}\right]$	M1	Correct expression or FT *their* **(b)**
	$\frac{20}{23}$ or 0.87[0]	A1	
		2	

Question	Answer	Marks	Guidance
5(a)	<table><tr><td>Class width</td><td>10</td><td>10</td><td>20</td><td>20</td><td>40</td></tr><tr><td>Frequency Density</td><td>1.6</td><td>5.4</td><td>3.9</td><td>1.6</td><td>0.5</td></tr></table>	M1	At least 4 frequency densities calculated, accept unsimplified. May be read from graph using *their* scale, 3SF or correct
		A1	All heights correct on graph
		B1	Bar ends at 0, 10, 20 …, etc. with a horizontal linear scale with at least 3 values indicated, 0 ⩽ horizontal axis ⩽ 100
		B1	Axes labelled: Frequency density (fd), time (t) and seconds. Linear vertical scale, with at least 3 values indicated 0 ⩽ vertical axis ⩽ 5.4
		4	

Question	Answer	Marks	Guidance
5(b)	$\text{Mean} = \left[\frac{16\times5+54\times15+78\times30+32\times50+20\times80}{200}\right]$ $= \frac{80+810+2340+1600+1600}{200}$	M1	Uses at least 4 midpoint attempts (e.g. 5 ± 0.5). Accept unsimplified expression, denominator either correct or *their* Σfrequencies
	$\left[\frac{6430}{200} =\right] 32\frac{3}{20}$ or 32.15	A1	Accept 32.2
		2	
5(c)	A value in correct UQ (40–60) – a value in correct LQ (10–20)	M1	
	Greatest possible value is 60 – 10 = 50	A1	Condone $49.\dot{9}$
		2	

Question	Answer	Marks	Guidance
6(a)	$1 - P(10, 11, 12) = 1 - ({}^{12}C_{10}\,0.6^{10}\,0.4^{2} + {}^{12}C_{11}\,0.6^{11}\,0.4^{1} + {}^{12}C_{12}\,0.6^{12}\,0.4^{0})$ [= 1 – (0.063852 + 0.017414 + 0.0021768)]	M1	One term: ${}^{12}C_x\,p^x\,(1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression, or better.
	[1 – 0.083443] = 0.917	A1	AWRT
	Alternative method for Question 6(a)		
	$P(0,1,2,3,4,5,6,7,8,9) = {}^{12}C_0 0.6^0\,0.4^{12} + {}^{12}C_1\,0.6^1\,0.4^{11} + \ldots\ldots\ldots\ldots {}^{12}C_9\,0.6^9\,0.4^3$ [= 0.000016777 + 0.00030199 + 0.0024914 + 0.012457 + 0.042043 + 0.10090 + 0.17658 + 0.22703 + 0.21284 + 0.14189]	M1	One term: ${}^{12}C_x\,p^x\,(1-p)^{12-x}$ for $0 < x < 12$, any p allowed.
		A1	Correct unsimplified expression with at least the first two and last terms
	0.917	A1	WWW, AWRT
		3	

Question	Answer	Marks	Guidance
6(b)	[Mean =] 0.6×150 [= 90]; [Variance =] $0.6 \times 150 \times 0.4$ [= 36]	B1	Correct mean and variance. Accept evaluated or unsimplified
	$P(X < 81) = P\left(Z < \frac{80.5 - 90}{6}\right)$	M1	Substituting *their* mean and variance into ±standardisation formula (with a numerical value for 80.5), allow σ^2, $\sqrt{\sigma}$, but not $\mu \pm 0.5$
		M1	Using continuity correction 80.5 or 81.5
	$\Phi(-1.5833) = 1 - 0.9433$	M1	Appropriate area Φ, from final process, must be probability
	0.0567	A1	AWRT
		5	
6(c)	$np = 90$, $nq = 60$ both greater than 5	B1	At least nq evaluated and statement >5 required
		1	

Question	Answer	Marks	Guidance
7(a)	$P(X = 3) = \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5}$	M1	$\frac{m}{7} \times \frac{n}{6} \times \frac{o}{5}$ used throughout. condone use of $\frac{1}{2}$
	$\frac{6}{35}$	A1	AG. The fractions must be identified, e.g. P(NC, NC, C), may be seen in a tree diagram.
		2	

Question	Answer	Marks	Guidance
7(b)	$\begin{array}{\|c\|c\|c\|c\|c\|c\|}\hline x & 1 & 2 & 3 & 4 & 5 \\ \hline p & \frac{15}{35} & \frac{10}{35} & \frac{6}{35} & \frac{3}{35} & \frac{1}{35} \\ \hline \end{array}$	B1	Table with x values and at least one probability Condone any additional x values if probability stated as 0.
		B1	One correct probability other than $X = 3$ linked to the correct outcome
		B1	Two further correct probabilities other than $X = 3$ seen linked to the correct outcome
		B1FT	All probabilities correct, or at least 4 probabilities summing to 1
		4	
7(c)	$[E(X) = 1\times\frac{15}{35} + 2\times\frac{10}{35} + 3\times\frac{6}{35} + 4\times\frac{3}{35} + 5\times\frac{1}{35}]$ $E(X) = \frac{15+20+18+12+5}{35}\left[= \frac{70}{35} = 2\right]$	M1	At least 4 correct terms FT *their* values in **(a)** with probabilities summing to 1 May be implied by use in Variance, accept unsimplified expression.
	$\text{Var}(X) = \left[\frac{1^2\times15 + 2^2\times10 + 3^2\times6 + 4^2\times3 + 5^2\times1}{35} - 2^2 = \right]$ $\frac{15+40+54+48+25}{35} - 2^2$	M1	Appropriate variance formula using *their* $(\text{E}(X))^2$. FT *their* table accept probabilities not summing to 1.
	$\left[= \frac{182}{35} - 4\right] = \frac{6}{5}$	A1	**N.B.** If method FT for M marks from *their* incorrect **(b)**, expressions for E(X) and Var(X) must be seen unsimplified with all probabilities <1
		3	