Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER			

2969009541

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2019**

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.

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1 Use the trapezium rule with 3 intervals to estimate the value of

$$\int_0^3 \left|2^x - 4\right| \mathrm{d}x. \qquad [3]$$

2 Showing all necessary working, solve the equation $\ln(2x - 3) = 2\ln x - \ln(x - 1)$. Give your answer correct to 2 decimal places. [4]

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3 Find the gradient of the curve $x^3 + 3xy^2 - y^3 = 1$ at the point with coordinates $(1, 3)$. [4]

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4 By first expressing the equation $\cot\theta - \cot(\theta + 45°) = 3$ as a quadratic equation in $\tan\theta$, solve the equation for $0° < \theta < 180°$. [6]

5 **(i)** Differentiate $\frac{1}{\sin^2\theta}$ with respect to θ. [2]

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(ii) The variables x and θ satisfy the differential equation

$$x\tan\theta\frac{\mathrm{d}x}{\mathrm{d}\theta} + \mathrm{cosec}^2\theta = 0,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 4$ when $\theta = \frac{1}{6}\pi$. Solve the differential equation, obtaining an expression for x in terms of θ. [6]

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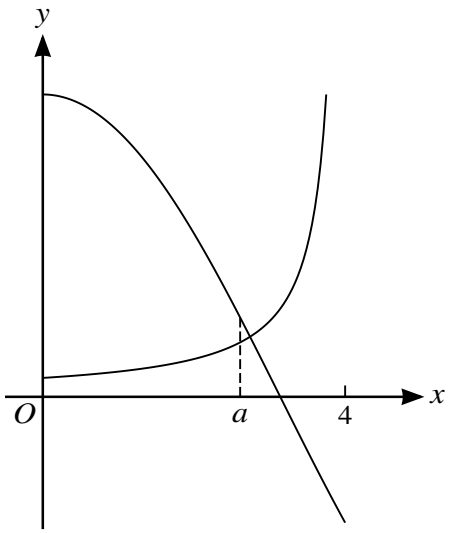
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6 **(i)** By first expanding $\sin(2x + x)$, show that $\sin 3x \equiv 3 \sin x - 4 \sin^3 x$. [4]

(ii) Hence, showing all necessary working, find the exact value of $\int_0^{\frac{1}{3}\pi} \sin^3 x \, \mathrm{d}x$. [4]

7



The diagram shows the curves $y = 4\cos\frac{1}{2}x$ and $y = \dfrac{1}{4-x}$, for $0 \leqslant x < 4$. When $x = a$, the tangents to the curves are perpendicular.

(i) Show that $a = 4 - \sqrt{(2\sin\frac{1}{2}a)}$. [4]

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(ii) Verify by calculation that a lies between 2 and 3. [2]

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(iii) Use an iterative formula based on the equation in part **(i)** to determine a correct to 3 decimal places. Give the result of each iteration to 5 decimal places. [3]

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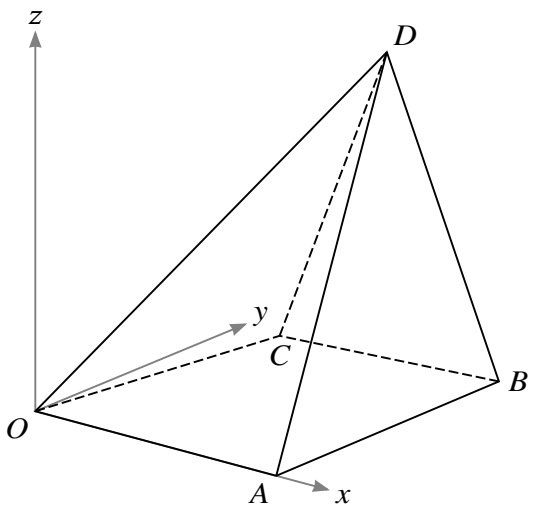
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8 Let $\mathrm{f}(x) = \dfrac{16 - 17x}{(2 + x)(3 - x)^2}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Hence obtain the expansion of $f(x)$ in ascending powers of x, up to and including the term in x^2. [5]

9



The diagram shows a set of rectangular axes Ox, Oy and Oz, and four points A, B, C and D with position vectors $\overrightarrow{OA} = 3\mathbf{i}$, $\overrightarrow{OB} = 3\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OC} = \mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OD} = 2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}$.

(i) Find the equation of the plane BCD, giving your answer in the form $ax + by + cz = d$. [6]

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(ii) Calculate the acute angle between the planes BCD and $OABC$. [4]

10 Throughout this question the use of a calculator is not permitted.

The complex number $(\sqrt{3}) + \mathrm{i}$ is denoted by u.

(i) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$, giving the exact values of r and θ. Hence or otherwise state the exact values of the modulus and argument of u^4. [5]

(ii) Verify that u is a root of the equation $z^3 - 8z + 8\sqrt{3} = 0$ and state the other complex root of this equation. [3]

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(iii) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leqslant 2$ and $\mathrm{Im}\, z \geqslant 2$, where $\mathrm{Im}\, z$ denotes the imaginary part of z. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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