Cambridge International AS & A Level

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MATHEMATICS **9709/13**

Paper 1 **May/June 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question	Answer	Marks	Guidance
1(i)	$\left[(x-2)^2\right]$ [+4]	**B1 DB1**	2nd B1 dependent on 2 inside bracket
		2	
1(ii)	$(x-2)^2 < 5 \rightarrow -\sqrt{5} < x-2$ and/or $x-2 < \sqrt{5}$	**M1**	Allow e.g. $x-2 < \pm\sqrt{5}$, $x-2 = \pm\sqrt{5}$ and decimal equivalents for √5 For M1, ft from *their*(i). Also allow √13 instead of √5 for clear slip
	$2-\sqrt{5} < x < 2+\sqrt{5}$	**A1A1**	A1 for each inequality – allow two separate statements but there must be 2 inequalities for x. Non-hence methods, if completely correct, score SC 1/3. Condone $\leqslant$
		[3]	

Question	Answer	Marks	Guidance
2(i)	$\frac{-5}{x}+\frac{5}{8x^3}-\frac{1}{32x^5}$ (or $-5x^{-1}+\frac{5}{8}x^{-3}-\frac{1}{32}x^{-5}$)	**B1B1B1**	B1 for each correct term SCB1 for both $\frac{+5}{x}$ & $\frac{+1}{32x^5}$
		3	
2(ii)	$1\times 20 + 4\times \textit{their}(-5) = 0$	**M1A1**	Must be from exactly 2 terms SCB1 for $20+20=40$
		2	

Question	Answer	Marks	Guidance
3(i)	Angle EAD = Angle $ACD = \frac{3\pi}{10}$ or 54° or 0.942 soi **or** Angle $DAC = \frac{\pi}{5}$ or 36° or 0.628 soi	**B1**	
	$AD = 8\sin(\frac{3\pi}{10})$ **or** $8\cos(\frac{\pi}{5})$	**M1**	Angles used must be correct
	(AD =) 6.47	**A1**	
	Alternative method for question 3(i)		
	$AB = \frac{8}{\tan\left(\frac{\pi}{5}\right)}$ or $AB = \frac{8\sin\left(\frac{3\pi}{10}\right)}{\sin\left(\frac{\pi}{5}\right)}$ or $11.(01)$	**B1**	Angles used must be correct
	$AD = 11.0(1)\sin\frac{\pi}{5}$ **oe**	**M1**	
	(AD =) 6.47	**A1**	
		3	
3(ii)	Area sector $= \frac{1}{2}(theirAD)^2 \times their\left(\frac{\pi}{2} - \frac{\pi}{5}\right)$	**M1**	19.7(4)
	Area $\Delta ADC = \frac{1}{2} \times 8 \times theirAD \times \sin\frac{\pi}{5}$ or $\frac{1}{2} \times 8\cos\left(\frac{3\pi}{10}\right) \times 8\sin\left(\frac{3\pi}{10}\right)$	**M1**	Or e.g. ½ $theirAD \times \sqrt{8^2 - theirAD^2}$. 15.2(2)
	(Shaded area =) 35.0 or 34.9	**A1**	
		3	

Question	Answer	Marks	Guidance
4(i)	Max(***a***) is 8	**B1**	Allow $a = 8$ or $a \leqslant 8$
	Min(***b***) is 24	**B1**	Allow $b = 24$ or $b \geqslant 24$
		2	SCB1 for 8 and 24 seen
4(ii)	$\text{gf}(x) = \frac{96}{x-1} - 4$ **or** $\text{gf}(x) = \frac{100-4x}{x-1}$	**B1**	$2\left(\frac{48}{x-1}\right) - 4$ is insufficient Apply ISW
		1	
4(iii)	$y = \frac{96}{x-1} - 4 \rightarrow y + 4 = \frac{96}{x-1} \rightarrow x - 1 = \frac{96}{y+4}$	**M1**	**FT** from *their*(**ii**) provided **(ii)** involves algebraic fraction. Allow sign errors
	$(\text{gf})^{-1}(x) = \frac{96}{x+4} + 1$	**A1**	OR $\frac{100+x}{x+4}$. Must be a function of x. Apply ISW
		2	

Question	Answer	Marks	Guidance
5(i)	$\frac{x}{2}\left[2 + (x-1)(-/+0.02)\right]$ or $1.01x - 0.01x^2$ or $0.99x + 0.01x^2$ **oe**	**B1**	Allow – or + 0.02. Allow n used
		1	

Question	Answer	Marks	Guidance
5(ii)	Equate to 13 **then** *either* simplify to a 3-term quadratic equation **or** find at least 1 solution (need not be correct) to an unsimplified quadratic	**M1**	Expect $n^2 - 101n + 1300$ (=0) or $0.99x + 0.01x^2 = 13$. Allow x used
	16	**A1**	Ignore 85.8 or 86
		2	
5(iii)	Use of $\frac{a(1-r^n)}{1-r}$ with $a = 1$, $r = 0.92$, $n = 20$ soi	**M1**	
	(=) 10.1	**A1**	
	Use of $(S_\infty =)\frac{a}{1-r}$ with $a = 1$, $r = 0.92$	**M1**	OR $\frac{(1)(1-0.92^n)}{1-0.92} = 13 \rightarrow 0.92^n = -0.04$ oe
	$S_\infty = 12.5$ so never reaches target or < 13	**A1**	Conclusion required – 'Shown' is insufficient No solution so never reaches target or < 13
		4	

Question	Answer	Marks	Guidance
6(i)	$\mathbf{MF} = -4\mathbf{i} + 2\mathbf{j} + 7\mathbf{k}$	**B1**	
		1	
6(ii)	$\mathbf{FN} = 2\mathbf{i} - \mathbf{j}$	**B1**	
		1	
6(iii)	$\mathbf{MN} = -2\mathbf{i} + \mathbf{j} + 7\mathbf{k}$	**B1**	**FT** on *their* (**MF** + **FN**)
		1	

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Question	Answer	Marks	Guidance
6(iv)	**MF.MN** = 8 + 2 + 49 = 59	***M1**	**MF.MN** or **FM.NM** but allow if one is reversed (implied by –59)
	$\|\mathbf{MF}\| \times \|\mathbf{MN}\| = \sqrt{4^2+2^2+7^2} \times \sqrt{2^2+1^2+7^2}$	***DM1**	Product of modulus. At least one methodically correct
	$\cos FMN = \dfrac{+/-59}{\sqrt{69} \times \sqrt{54}}$	**DM1**	All linked correctly. Note $\sqrt{69} \times \sqrt{54} = 9\sqrt{46}$
	$FMN = 14.9°$ or 0.259	**A1**	Do not allow if exactly 1 vector is reversed – even if adjusted finally
		4	

Question	Answer	Marks	Guidance
7(i)	$D = (5, 1)$	**B1**	
		1	
7(ii)	$(x-5)^2 + (y-1)^2 = 20$ **oe**	**B1**	**FT** on *their D*. Apply ISW, oe but not to contain square roots
		1	

Question	Answer	Marks	Guidance
7(iii)	$(x-1)^2+(y-3)^2=(9-x)^2+(y+1)^2$ **soi**	**M1**	Allow 1 sign slip For M1 allow with √ signs round both sides but sides must be equated
	$x^2-2x+1+y^2-6y+9=x^2-18x+81+y^2+2y+1$	**A1**	
	$y=2x-9$ www **AG**	**A1**	
	Alternative method for question 7(iii)		
	grad. of $AB=-½ \rightarrow$ grad of perp bisector $=\frac{-1}{-½}$	**M1**	
	Equation of perp. bisector is $y-1=2(x-5)$	**A1**	
	$y=2x-9$ www **AG**	**A1**	
		3	
7(iv)	Eliminate y (or x) using equations in **(ii)** and **(iii)**	***M1**	To give an (unsimplified) quadratic equation
	$5x^2-50x+105\ (=0)$ **or** $5(x-5)^2=20$ **or** $5y^2-10y-75\ (=0)$ **or** $5(y-1)^2=80$	**DM1**	Simplify to one of the forms shown on the right (allow arithmetic slips)
	$x=3$ and 7, **or** $y=-3$ and 5	**A1**	
	(3, –3), (7, 5)	**A1**	Both pairs of x & y correct implies A1A1. SC B2 for no working
		4	

Question	Answer	Marks	Guidance
8	$f'(-1)=0 \rightarrow 3-a+b=0$ $f'(3)=0 \rightarrow 27+3a+b=0$	**M1**	Stationary points at $x=-1$ & $x=3$ gives sim. equations in a & b
	$a=-6$	**A1**	Solve simultaneous equation
	$b=-9$	**A1**	
	Hence $f'(x)=3x^2-6x-9 \rightarrow f(x)=x^3-3x^2-9x(+c)$	**B1**	**FT** correct integration for *their a,b* (numerical a, b)
	$2=-1-3+9+c$	**M1**	Sub $x=-1$, $y=2$ into *their* integrated f(x). c must be present
	$c=-3$	**A1**	**FT** from *their* f(x)
	$f(3)=k \rightarrow k=27-27-27-3$	**M1**	Sub $x=3$, $y=k$ into *their* integrated f(x) (Allow c omitted)
	$k=-30$	**A1**	
		8	

Question	Answer	Marks	Guidance
9(i)	$q \leqslant f(x) \leqslant p+q$	**B1B1**	B1 each inequality – allow two separate statements Accept $<$, $(q, p+q)$, $[q, p+q]$ Condone y or x or f in place of f(x)
		2	

Question	Answer	Marks	Guidance
9(ii)	**(a)** 2	**B1**	Allow $\frac{\pi}{4}, \frac{3\pi}{4}$
	(b) 3	**B1**	Allow $0, \frac{\pi}{2}, \pi$
	(c) 4	**B1**	Allow $\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$
		3	
9(iii)	$3\sin^2 2x + 2 = 4 \rightarrow \sin^2 2x = \frac{2}{3}$ soi	**M1**	
	$\text{Sin}2x = (\pm)0.816(5)$. Allow $\sin 2x = (\pm)\sqrt{\frac{2}{3}}$ or $2x = \sin^{-1}(\pm)\sqrt{\frac{2}{3}}$	**A1**	OR Implied by at least one correct value for x. Allow $\sin^{-1}$ form
	$(2x =)$ **at least two** of 0.955(3), 2.18(6), 4.09(7) , 5.32(8)	**A1**	Can be implied by corresponding values of x below Allow for at least two of 0.304π, 0.696π, 1.30(4)π, 1.69(6)π **OR** at least <u>two</u> of 54.7(4)°, 125.2(6)°, 234.7(4)°, 305.2(6)°
	$(x =)$ 0.478, 1.09, 2.05, 2.66.	**A1A1**	Allow 0.152π, 0.348π, 0.652π, 0.848π SC A1 for 2 or 3 correct. SC A1 for all of 27.4º, 62.6º, 117.4º, 152.6º $\text{Sin}2x = \pm\frac{2}{3} \rightarrow x = 0.365, 1.21, 1.94, 2.78$ scores SC M1A0A0A1
		5	

Question	Answer	Marks	Guidance
10(i)	$\left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right]$	**B1**	oe
	$\frac{dy}{dx}=\left[\frac{1}{2}(3x+4)^{-\frac{1}{2}}\right]\times 3$	**B1**	Must have '$\times 3$'
	At $x=4$, $\frac{dy}{dx}=\frac{3}{8}$ soi	**B1**	
	Line through (4, *their*4) with gradient *their* $\frac{3}{8}$	**M1**	If $y\neq 4$ is used then clear evidence of substitution of $x=4$ is needed
	Equation of tangent is $y-4=\frac{3}{8}(x-4)$ or $y=\frac{3}{8}x+\frac{5}{2}$	**A1**	oe
		5	

Question	Answer	Marks	Guidance
10(ii)	Area under line $= \frac{1}{2}\left(4+\frac{5}{2}\right)\times 4 = 13$	**B1**	OR $\int_0^4 \frac{3}{8}x+\frac{5}{2} = \left[\frac{3}{16}x^2+\frac{5}{2}x\right] = [3+10] = 13$
	Area under curve: $\int (3x+4)^{\frac{1}{2}} = \left[\frac{(3x+4)^{3/2}}{3/2}\right]\left[\div 3\right]$	**B1B1**	Allow if seen as part of the difference of 2 integrals First B1 for integral without $\left[\div 3\right]$ Second B1 must have $\left[\div 3\right]$
	$\frac{128}{9}-\frac{16}{9} = \frac{112}{9} = 12\frac{4}{9}$	**M1**	Apply limits 0 → 4 to an integrated expression
	Area $= 13 - 12\frac{4}{9} = \frac{5}{9}$ (or 0.556)	**A1**	
	Alternative method for question 10(ii)		
	Area for line = 1/2 × 4 × 3/2 = 3	**B1**	OR $\int_{5/2}^4 \frac{1}{3}(8y-20) = \frac{1}{3}\left[4y^2-20\right] = \frac{1}{3}[-16+25] = 3$
	Area for curve = $\int \frac{1}{4}(y^2-4) = \left[\frac{y^3}{9}\right]-\left[\frac{4y}{3}\right]$	**B1B1**	
	$\left(\frac{64}{9}-\frac{16}{3}\right)-\left(\frac{8}{9}-\frac{8}{3}\right) = \frac{32}{9}$	**M1**	Apply limits 2 → 4 to an integrated expression for curve
	Area $= \frac{32}{9}-3 = \frac{5}{9}$ (or 0.556)	**A1**	
		5	

Question	Answer	Marks	Guidance
10(iii)	$\frac{dy}{dx} = \frac{1}{2}$	**B1**	
	$\frac{3}{2}(3x+4)^{-\frac{1}{2}} = \frac{1}{2}$	**M1**	Allow M1 for $\frac{3}{2}(3x+4)^{-\frac{1}{2}} = 2$.
	$(3x+4)^{\frac{1}{2}} = 3 \rightarrow 3x+4=9 \rightarrow x = \mathbf{\frac{5}{3}}$ oe	**A1**	
		3	