Cambridge International AS & A Level

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MATHEMATICS **9709/63**

Paper 6 **May/June 2018**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously 'correct' answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

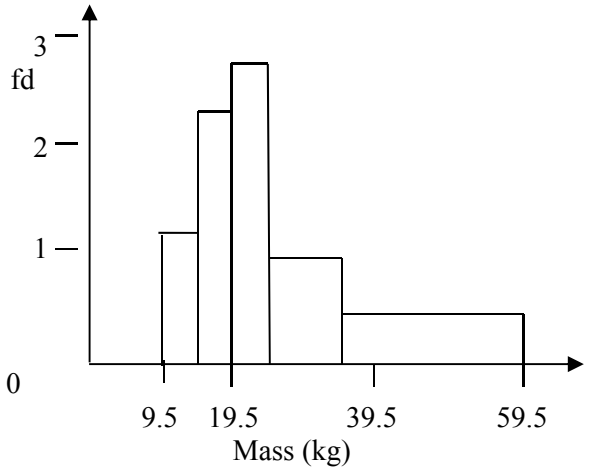
SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become 'follow through' marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question	Answer	Marks	Guidance
1(i)	15–19 (kg) cao	**B1**	kg not necessary; condone 14.5 – 19.5
	Total:	**1**	
1(ii)	fd = 1.2, 2.4, 2.8, 1, 0.32  	**M1**	Attempt at fd [f/(attempt at cw)] or scaled freq (may be implied by 4 correct)
		A1	Correct heights seen on diagram with linear vertical scale from (x, 0)
		B1	Correct bar widths (1:1:1:2:5) visually no gaps with linear horizontal scale from (9.5,y) and first bar starting at (9.5, y)
		B1	Histogram, using attempted fds, with labels (mass, kg and fd seen) and at least 3 linearly spaced values on each axis. Horizontal axis must range from at least 9.5 to 59.5 If horizontal axis clearly starts from zero, either a break in the scale must be indicated or the scale must be linear from zero.

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Question	Answer	Marks	Guidance
2(i)	$z = 0.674$	**B1**	z value ±0.674
	$0.674 = \dfrac{0 - -3}{\sigma}$	**M1**	±Standardising with 0 and equating to a z-value
	$\sigma = 4.45$	**A1**	Correct answer www ie not ignoring a minus sign
	Total:	**3**	
2(ii)	P(0, 1)	**M1**	Any bin of form ${}^8C_x(0.75)^x\,(0.25)^{8-x}$ any x
	$= (0.75)^8 + {}^8C_1(0.25)(0.75)^7$	**M1**	Correct unsimplified answer, may be implied by numerical values
	$0.1001 + 0.2670 \quad = 0.367$	**A1**	Correct answer
	Method 2 $1 - P(8,7,6,5,4,3,2) = 1 - (0.25)^8 - {}^8C_1(0.75)(0.25)^7 - \ldots$	**M1**	Any bin of form ${}^8C_x(0.75)^x\,(0.25)^{8-x}$ any x
	$- {}^8C_2(0.75)^6\,(0.25)^2$	**M1**	Correct unsimplified answer
	$= 0.367$	**A1**	Correct answer
	Total:	**3**	

Question	Answer	Marks	Guidance
3(i)	$(1-x)$ and 0.45 (or 0.3)	**B1**	Seen, either on tree diagram or elsewhere
	Beginners: $0.7 \times x$ + '0.45' × '$(1-x)$' = 0.5 Or Advanced: '0.3' × x + 0.55 × '$(1-x)$' = 0.5 Or $0.7 \times x$ + '0.45'× '$(1-x)$' = '0.3' × x + 0.55× '$(1-x)$'	**M1**	One of the three correct probability equations
	$x = 0.2$ oe	**A1**	Correct answer
	Total:	**3**	
3(ii)	$P(M \mid A) = \frac{P(M \cap A)}{P(A)} = \frac{0.2 \times 0.3}{0.5}$	**M1**	'i' × 0.3 as num or denom of a fraction
		M1	0.5 (or (1 – 'i') × 0.55 + 'i' × 0.3 unsimplified) seen as denom of a fraction
	$= 0.12 \left(\frac{3}{25}\right)$	**A1**	Correct answer
	Total:	**3**	

Question	Answer	Marks	Guidance
4(i)	Mean = $(30 \times 1500 + 21 \times 2400)/51$	**M1**	Multiply by 30 and 21, summing and dividing total by 51 $\left(\frac{45000+50400}{51}\right)$
	= 1870 (1870.59)	**A1**	correct answer (to 3sf)
	Total:	**2**	
4(ii)	$230^2 = \frac{\Sigma x_F{}^2}{30} - 1500^2$ so $\Sigma x_F{}^2 = 69\,087\,000$	**M1**	One correct substitution into a correct variance formula
		A1	Correct $\Sigma x_F{}^2$ (rounding to 69 000 000 2sf)
	$160^2 = \frac{\Sigma x_L{}^2}{21} - 2400^2$ so $\Sigma x_L{}^2 = 121\,497\,600$	**A1**	Correct $\Sigma x_L{}^2$ (rounding to 121 000 000 3sf)
	New var = $\frac{69\,087\,000 + 121\,497\,600}{51} - 1870.588^2 = 237\,853$	**M1**	using '$\Sigma x_F{}^2$'+ '' $\Sigma x_L{}^2$ dividing by 51 and subtracting 'i' squared. (Correct '$\Sigma x_F{}^2$' + '' $\Sigma x_L{}^2$ = 190 584 600)
	New sd = 488	**A1**	Correct answer accept anything between 486 and 490
	Total:	**5**	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="6">5(i)</td><td>P(0) = 0.6 × 0.25 × 0.5 = 0.075
P(1) = 0.4 × 0.25 × 0.5 + 0.6 × 0.75 × 0.5 + 0.6 × 0.25 × 0.5 = 0.35
P(2) = 0.4 × 0.75 × 0.5 + 0.4 × 0.25 × 0.5 + 0.6 × 0.75 × 0.5 = 0.425
P(3) = 0.4 × 0.75 × 0.5 = 0.15</td><td>B1</td><td>0, 1, 2, 3 seen as top line of a pdf table OR attempting to evaluate P(0), P(1), P(2) and P(3)</td></tr>
<tr><td></td><td>M1</td><td>Multiply 3 probabilities together from 0.4 or 0.6, 0.25 or 0.75, 0.5 with or without a table</td></tr>
<tr><td rowspan="3"><table><tr><td>No of heads</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>Prob</td><td>0.075 $\left(\frac{3}{40}\right)$</td><td>0.35 $\left(\frac{7}{20}\right)$</td><td>0.425 $\left(\frac{17}{40}\right)$</td><td>0.15 $\left(\frac{3}{20}\right)$</td></tr></table></td><td>M1</td><td>Summing 3 probabilities for P(1) or P(2) with or without a table</td></tr>
<tr><td>B1</td><td>One correct probability seen.</td></tr>
<tr><td>A1</td><td>All correct in a table</td></tr>
<tr><td></td><td>Total:</td><td>5</td><td></td></tr>
<tr><td>5(ii)</td><td>E(X) = 0.35 + 2 × 0.425 + 3 × 0.15 = 1.65 $\left(\frac{33}{20}\right.$ oe$\left.\right)$</td><td>M1</td><td>Correct unsimplified expression for the mean using their table, ∑p = 1; can be implied by correct answer</td></tr>
<tr><td rowspan="3">5(ii)</td><td>Var(X) = 0.35 + 4 × 0.425 + 9 × 0.15 – 1.65^2</td><td>M1</td><td>Correct unsimplified expression for the variance using their table and their mean^2 subtracted, ∑p = 1</td></tr>
<tr><td>= 0.678 (0.6775) $\left(\frac{271}{400}\right.$ oe$\left.\right)$</td><td>A1</td><td>Correct answer</td></tr>
<tr><td>Total:</td><td>3</td><td></td></tr>
</table>

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Question	Answer	Marks	Guidance
6(i)	$z_1 = \pm\dfrac{4.1-5.7}{0.8} = -2 \qquad z_2 = \pm\dfrac{5-5.7}{0.8} = -0.875$	**M1**	At least one standardising no cc no sq rt no sq using 5.7 and 0.8 and either 4.1 or 5
	P(Toffee Apple) $= P(d < 5.0) - P(d < 4.1)$ $= P(z < -0.875) - P(z < -2)$ $= \Phi(-0.875) - \Phi(-2)$ $= \Phi(2) - \Phi(0.875)$	**M1**	Correct area $\Phi - \Phi$ legitimately obtained – need 2 negative z-values or 2 positives – not one of each
	$= 0.9772 - 0.8092 = 0.168$ (or $0.1908 - 0.0228$)	**A1**	Correct final answer
	Total:	**3**	
6(ii)	$np = 250 \times 0.168 = 42, \qquad npq = 34.944$	**B1ft**	Correct unsimplified mean and var – ft their prob for **(i)** providing $(0 < p < 1)$ Implied by $\sigma = \sqrt{34.944} = 5.911$
	$P(< 50) = P\left(z < \dfrac{49.5-42}{\sqrt{34.944}}\right) = P(z < 1.2687)$	**M1**	± Standardising using 50, their mean and sd; must have sq rt.
		M1	49.5 or 50.5 seen as a cc
	$= \Phi(1.2687)$	**M1**	Correct area $\Phi(> 0.5$ for + z and < 0.5 for –z)in their final answer
	$= 0.898$	**A1**	Correct final answer
	Total:	**5**	

Question	Answer	Marks	Guidance
7(i)	****E**** Other letters arranged in $\frac{8!}{2!3!}$ = 3360 ways	**M1**	Mult by 8! or ${}^{8}P_{8}$ oe (arrangements ignoring repeats)
		A1	Correct final answer www
	OR $\frac{8\times7\times6\times5\times4\times4\times3\times2\times1}{4!2!}$ = 3360 ways	**M1**	Correct numerator (161 280)
		A1	Correct final answer www
	Total:	**2**	
7(ii)	* * * * * ↑ Arrangements other letters × ways Es inserted $= \frac{5!}{2!} \times {}^{6}C_{4}\left(\frac{5!}{2!}\times\frac{{}^{6}P_{4}}{4!}\right)$	**M1**	k mult by ${}^{6}C_{4}$ or ${}^{6}P_{4}$ oe (ways to insert Es ignoring repeats), k can = 1 or k mult by $\frac{5!}{2!}$
		M1	Correct unsimplified expression or $\frac{5!}{2!} \times {}^{6}P_{4}$
	= 900 ways	**A1**	Correct answer
	OR Total no of ways – no of ways with Es touching 9!/(4! × 2!) – … or 7 560 – … $\frac{6!}{2!} + {}^{6}P_{2}\times\frac{5!}{2!} + \frac{{}^{6}P_{2}}{2!}\times\frac{5!}{2!} + \frac{{}^{6}P_{3}}{2!\times\frac{5!}{2!}}$ = 360 + 1800 + 900 + 3600 = 6660	**M1**	7560 unsimplified – k
		M1	Attempting to find four ways of Es touching (4 Es, 3Es and a single, 2 lots of 2 Es, 2 Es and 2 singles)
	7 560 – 6 660 = 900	**A1**	Correct answer

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Question	Answer	Marks	Guidance
7(ii)	OR Adding the number of ways with the first E in the 1st (E_1), 2nd (E_2) or 3rd (E_3) position. $\frac{5!}{2!}(E_1 + E_2 + E_3)$ where $E_1 = 10$, $E_2 = 4$, $E_3 = 1$ $\frac{5!}{2!}(E_1 + E_2 + E_3)$	**M1**	For any values for E_1, E_2 and E_3
		M1	For any two correct values of E_1, E_2 and E_3
	600 + 240 + 60 = 900	**A1**	Correct answer
	Total:	**3**	
7(iii)	EENN* in 3 ways	**B1**	Numerical value must be stated
	Total:	**1**	

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Question	Answer	Marks	Guidance
7(iv)	EE *** with no N: 1 way EEN** 3C2 or listing 3 ways EENN* 3 ways from **(iii)**	**M1**	Identifying the three different scenarios of EE, EEE or EEEE
		A1	Total no of ways with two Es (7 or 3 + 3 + 1)
	EEE** with no N: 3 ways EEEN* 3 ways EEENN 1 way	**A1**	Total no. of ways with 3 Es (7)
	EEEE* no N 3 ways EEEEN 1 way Total 18 ways	**A1**	Correct answer stated
	Method List containing ways with 2Es, 3Es and 4Es List containing at least 8 correct different ways List of all 18 correct ways Total 18	**M1**	At least 1 option listed for each of EE^^^, EEE^^, EEEE^
		A1	Ignore repeated options
		A1	Ignore repeated/incorrect options
		A1	Correct answer stated
	Total:	**4**	

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