Cambridge International A Level

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MATHEMATICS **9709/31**

Paper 3 **May/June 2016**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1 (**i**) *EITHER*: State or imply non-modular equation $(2(x-1))^2 = (3x)^2$, or pair of linear equations $2(x-1) = \pm 3x$ **B1**

Make reasonable solution attempt at a 3-term quadratic, or solve two linear equations **M1**

Obtain answers $x = -2$ and $x = \frac{2}{5}$ **A1**

OR: Obtain answer $x = -2$ by inspection or by solving a linear equation (**B1**

Obtain answer $x = \frac{2}{5}$ similarly **B2)**

[3]

(**ii**) Use correct method for solving an equation of the form $5^x = a$ or $5^{x+1} = a$, where $a > 0$ **M1**

Obtain answer $x = -0.569$ only **A1**

[2]

2 Integrate by parts and reach $axe^{-2x} + b\int e^{-2x}\,dx$ **M1**

Obtain $-\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$, or equivalent **A1**

Complete the integration correctly, obtaining $-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$, or equivalent **A1**

Use limits $x = 0$ and $x = \frac{1}{2}$ correctly, having integrated twice **M1**

Obtain answer $\frac{1}{4} - \frac{1}{2}e^{-1}$, or exact equivalent **A1**

[5]

3 Correctly restate the equation in terms of $\sin\theta$ and $\cos\theta$ **B1**

Using Pythagoras obtain a horizontal equation in $\cos\theta$ **M1**

Reduce the equation to a correct quadratic in $\cos\theta$, e.g. $3\cos^2\theta - \cos\theta - 2 = 0$ **A1**

Solve a 3-term quadratic for $\cos\theta$ **M1**

Obtain answer $\theta = 131.8°$ only **A1**

[5]

[Ignore answers outside the given interval.]

4 Separate variables and attempt integration of at least one side **M1***

Obtain term $\ln y$ **A1**

Obtain terms $\ln x - x^2$ **A1**

Use $x = 1$ and $y = 2$ to evaluate a constant, or as limits **DM1***

Obtain correct solution in any form, e.g. $\ln y = \ln x - x^2 + \ln 2 + 1$ **A1**

Obtain correct expression for y, free of logarithms, i.e. $y = 2x\exp(1 - x^2)$ **A1**

[6]

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5 Use product rule **M1**
Obtain correct derivative in any form, e.g. $\cos x \cos 2x - 2\sin x \sin 2x$ **A1**
Equate derivative to zero and use double angle formulae **M1**
Remove factor of cos x and reduce equation to one in a single trig function **M1**
Obtain $6\sin^2 x = 1$, $6\cos^2 x = 5$ or $5\tan^2 x = 1$ **A1**
Solve and obtain $x = 0.421$ **A1**
[6]

[Alternative: Use double angle formula M1.Use chain rule to differentiate M1. Obtain correct derivative
e.g. $\cos\theta - 6\sin^2\theta\cos\theta$ A1, then as above.]

6 (**i**) Make recognizable sketch of a relevant graph **B1**
Sketch the other relevant graph and justify the given statement **B1**
[2]

(**ii**) State $x = \frac{1}{2}\ln(25/x)$ **B1**
Rearrange this in the form $5e^{-x} = \sqrt{x}$ **B1**
[2]

(**iii**) Use the iterative formula correctly at least once **M1**
Obtain final answer 1.43 **A1**
Show sufficient iterations to 4 d.p. to justify 1.43 to 2 d.p., or show there is a sign change in the
interval (1.425, 1.435) **A1**
[3]

7 (**i**) State or imply $6xy + 3x^2\frac{dy}{dx}$ as derivative of $3x^2y$ **B1**
State $3y^2\frac{dy}{dx}$ as derivative of y^3 **B1**
Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$ **M1**
Obtain the given answer **A1**
[4]

(**ii**) Equate numerator to zero **M1***
Obtain $x = 2y$, or equivalent **A1**
Obtain an equation in x or y **DM1***
Obtain the point (–2, –1) **A1**
State the point (0, 1.44) **B1**
[5]

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8 (**i**) State or imply the form $\frac{A}{x+1}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$ **B1**

Use a correct method to determine a constant **M1**
Obtain one of the values $A = 1$, $B = 3$, $C = 12$ **A1**
Obtain a second value **A1**
Obtain a third value **A1**
[5]

[Mark the form $\frac{A}{x+1}+\frac{Dx+E}{(x-3)^2}$, where $A = 1$, $D = 3$, $E = 3$, B1M1A1A1A1 as above.]

(**ii**) Use correct method to find the first two terms of the expansion of $(x+1)^{-1}, (x-3)^{-1}, (1-\frac{1}{3}x)^{-1}$,

$(x-3)^{-2}$, or $(1-\frac{1}{3}x)^{-2}$ **M1**
Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction **A1√ + A1√ + A1√**
Obtain final answer $\frac{4}{3}-\frac{4}{9}x+\frac{4}{3}x^2$, or equivalent **A1**
[5]

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9 (**i**) *EITHER*: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AB} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ **B1**

Use scalar product to obtain an equation in a, b, c e.g. $a - 2b - 3c = 0$, $a + b - c = 0$, or $3b + 2c = 0$ **M1**

State two correct equations **A1**

Solve to obtain ratio $a : b : c$ **M1**

Obtain $a : b : c = 5 : -2 : 3$ **A1**

Obtain equation $5x - 2y + 3z = 5$, or equivalent **A1**

*OR*1: Substitute for two points, e.g. A and B, and obtain $a + 3b + 2c = d$ and $2a + b - c = d$ (**B1**

Substitute for another point, e.g. C, to obtain a third equation and eliminate one unknown entirely from all three equations **M1**

Obtain two correct equations in three unknowns, e.g. in a, b, c **A1**

Solve to obtain their ratio **M1**

Obtain $a : b : c = 5 : -2 : 3$, $a : c : d = 5 : 3 : 5$, $a : b : d = 5 : -2 : 5$, or $b : c : d = -2 : 3 : 5$ **A1**

Obtain equation $5x - 2y + 3z = 5$, or equivalent **A1)**

*OR*2: Obtain a vector parallel to the plane, e.g. $\overrightarrow{AC} = \mathbf{i} + \mathbf{j} - \mathbf{k}$ (**B1**

Obtain a second such vector and calculate their vector product, e.g. $(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) \times (\mathbf{i} + \mathbf{j} - \mathbf{k})$ **M1**

Obtain two correct components of the product **A1**

Obtain correct answer e.g. $5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ **A1**

Substitute in $5x - 2y + 3z = d$ to find d **M1**

Obtain equation $5x - 2y + 3z = 5$, or equivalent **A1)**

*OR*3: Obtain a vector parallel to the plane, e.g. $\overrightarrow{BC} = 3\mathbf{j} + 2\mathbf{k}$ (**B1**

Obtain a second such vector and form correctly a 2-parameter equation for the plane **M1**

Obtain a correct equation, e.g. $\mathbf{r} = \mathbf{i} + 3\mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}) + \mu(3\mathbf{j} + 2\mathbf{k})$ **A1**

State three correct equations in x, y, z, λ, μ **A1**

Eliminate λ and μ **M1**

Obtain equation $3x - 2y + 3z = 5$, or equivalent **A1)**

[6]

(**ii**) Correctly form an equation for the line through D parallel to OA **M1**

Obtain a correct equation e.g. $\mathbf{r} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \lambda(\mathbf{i} + 3\mathbf{j} + 2\mathbf{k})$ **A1**

Substitute components in the equation of the plane and solve for λ **M1**

Obtain $\lambda = 2$ and position vector $-\mathbf{i} + 7\mathbf{j} + 6\mathbf{k}$ for P **A1**

Obtain the given answer correctly **A1**

[5]

10 (**a**) Square $x + iy$ and equate real and imaginary parts to 7 and $-6\sqrt{2}$ respectively **M1**

Obtain equations $x^2 - y^2 = 7$ and $2xy = -6\sqrt{2}$ **A1**

Eliminate one variable and find an equation in the other **M1**

Obtain $x^4 - 7x^2 - 18 = 0$ or $y^4 + 7y^2 - 18 = 0$, or 3-term equivalent **A1**

Obtain answers $\pm(3 - i\sqrt{2})$ **A1**

[5]

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(**b**) (**i**) Show point representing 1 + 2i **B1**
Show circle with radius 1 and centre 1 + 2i **B1**
Show a half line from the point representing 1 **B1**
Show line making the correct angle with the real axis **B1**
[4]

(**ii**) State or imply the relevance of the perpendicular from 1 + 2i to the line **M1**
Obtain answer $\sqrt{2}-1$ (or 0.414) **A1**
[2]

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