Cambridge International AS & A Level

Cambridge International Examinations
Cambridge International Advanced Subsidiary and Advanced Level

MATHEMATICS **9709/21**

Paper 2 **May/June 2016**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2016 series for most Cambridge IGCSE®, Cambridge International A and AS Level components and some Cambridge O Level components.

® IGCSE is the registered trademark of Cambridge International Examinations.

© UCLES 2016

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

© Cambridge International Examinations 2016

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a "fortuitous" answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

© Cambridge International Examinations 2016

1 Obtain first derivative of form $k_1 e^{4x} + \frac{k_2}{2x+3}$ **M1**

Obtain correct $12e^{4x} - \frac{12}{2x+3}$ **A1**

Obtain 8 **A1** [3]

2 Use $\cot\theta = 1 \div \tan\theta$ **B1**

Form equation involving $\tan\theta$ only and with no denominators involving θ **M1**

Obtain $\tan^2\theta = \frac{2}{7}$ **A1**

Obtain 28.1 **A1**

Obtain 151.9 **A1** [5]

Allow other valid methods

3 Rearrange to $3e^{2x} - 14e^x + 8 = 0$ or equivalent involving substitution **B1**

Solve quadratic equation in e^x to find two values of e^x ***M1**

Obtain $\frac{2}{3}$ and 4 **A1**

Use natural logarithms to solve equation of form $e^x = k$ where $k > 0$ dep on **DM1**

Allow M mark if left in exact form **M1**

Obtain -0.405 **A1**

Obtain 1.39 **A1** [6]

4 **(i)** Carry out division, or equivalent, at least as far as $8x^2 + kx$ **M1**

Obtain correct quotient $8x^2 + 14x - 15$ **A1**

Confirm remainder is 5 **A1** [3]

(ii) State or imply expression is $(x+2)(...\text{their quadratic quotient}...)$ **B1√**

Attempt factorisation of their quadratic quotient **M1**

Obtain $(x+2)(2x+5)(4x-3)$ **A1** [3]

(iii) State $\pm\frac{3}{4}$ and no others, following their 3 linear factors **B1√** [1]

5 **(i)** Obtain $\frac{dx}{d\theta} = 2\sec^2\theta$ and $\frac{dy}{d\theta} = 6\cos 2\theta$ **B1**

Use $\cos 2\theta = 2\cos^2\theta - 1$ or equivalent **B1**

Form expression for $\frac{dy}{dx}$ in terms of $\cos\theta$ **M1**

Confirm $6\cos^4\theta - 3\cos^2\theta$ with no errors seen **A1** [4]

(ii) Equate first derivative to zero and obtain at least $\cos\theta = \pm\frac{1}{\sqrt{2}}$ **B1**

Obtain $\theta = \frac{1}{4}\pi$ or equivalent **B1**

Obtain $(2, 3)$ **B1** [3]

(iii) State or imply $\theta = \frac{1}{3}\pi$ or equivalent **B1**

Obtain $-\frac{3}{8}$ or equivalent only **B1** [2]

© Cambridge International Examinations 2016

6 **(i)** Use quotient rule or equivalent ***M1**

Obtain $\frac{6x(x^2+4)-6x^3}{(x^2+4)^2}$ or equivalent **A1**

Equate first derivative to $\frac{1}{2}$ and remove algebraic denominators dep on *M1 **DM1**

Obtain $48p = p^4 + 8p^2 + 16$ or $48x = x^4 + 8x^2 + 16$ or equivalent **A1**

Confirm given result $p = \sqrt{\frac{48p-16}{p^2+8}}$ **A1** [5]

(ii) Consider sign of $p - \sqrt{\frac{48p-16}{p^2+8}}$ at 2 and 3 or equivalent **M1**

Complete argument correctly with appropriate calculations **A1** [2]

(iii) Carry out iteration process correctly at least once **M1**

Obtain final answer 2.728 **A1**

Show sufficient iterations to justify accuracy to 4 sf or show sign change in interval $(2.7275, 2.7285)$ **B1** [3]

7 **(a)** Rewrite integrand as $\sec^2 2x + \cos^2 2x$ **B1**

Express $\cos^2 2x$ in form $k_1 + k_2 \cos 4x$ **M1**

State correct $\frac{1}{2} + \frac{1}{2}\cos 4x$ **A1**

Integrate to obtain at least terms involving $\tan 2x$ and $\sin 4x$ **M1**

Obtain $\frac{1}{2}\tan 2x + \frac{1}{2}x + \frac{1}{8}\sin 4x$, condoning absence of $+c$ **A1** [5]

(b) Integrate to obtain $2x + 2\ln(3x-2)$ **B1**

Show correct use of $p \ln k = \ln k^p$ law at least once, must be using $\ln(3x-2)$ **M1**

Show correct use of $\ln m - \ln n = \ln \frac{m}{n}$ law, must be using $\ln(3x-2)$ **M1**

Use or imply $20 = \ln(e^{20})$ **B1**

Obtain $\ln(16e^{20})$ **A1** [5]

© Cambridge International Examinations 2016