Cambridge International A Level

Cambridge International Examinations
Cambridge International Advanced Level

MATHEMATICS **9709/71**

Paper 7 Probability & Statistics 2 **(S2)** **May/June 2014**

1 hour 15 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

* 1 8 6 2 9 4 5 3 6 5 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 50.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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1 The masses, in grams, of apples of a certain type are normally distributed with mean 60.4 and standard deviation 8.2. The apples are packed in bags, with each bag containing 8 randomly chosen apples. The bags are checked by Quality Control and any bag containing apples with a total mass of less than 436 g is rejected. Find the proportion of bags that are rejected. [4]

2 A die is biased. The mean and variance of a random sample of 70 scores on this die are found to be 3.61 and 2.70 respectively. Calculate a 95% confidence interval for the population mean score. [5]

3 The lengths, in centimetres, of rods produced in a factory have mean μ and standard deviation 0.2. The value of μ is supposed to be 250, but a manager claims that one machine is producing rods that are too long on average. A random sample of 40 rods from this machine is taken and the sample mean length is found to be 250.06 cm. Test at the 5% significance level whether the manager's claim is justified. [5]

4 The proportion of people who have a particular gene, on average, is 1 in 1000. A random sample of 3500 people in a certain country is chosen and the number of people, $\boldsymbol{X}$, having the gene is found.

(i) State the distribution of $\boldsymbol{X}$ and state also an appropriate approximating distribution. Give the values of any parameters in each case. Justify your choice of the approximating distribution. [3]

(ii) Use the approximating distribution to find $\mathrm{P}(\boldsymbol{X} \leqslant 3)$. [2]

5 The score on one throw of a 4-sided die is denoted by the random variable $\boldsymbol{X}$ with probability distribution as shown in the table.

$\boldsymbol{x}$	0	1	2	3
$\mathrm{P}(\boldsymbol{X} = \boldsymbol{x})$	0.25	0.25	0.25	0.25

(i) Show that $\mathrm{Var}(\boldsymbol{X}) = 1.25$. [1]

The die is thrown 300 times. The score on each throw is noted and the mean, $\overline{\boldsymbol{X}}$, of the 300 scores is found.

(ii) Use a normal distribution to find $\mathrm{P}(\overline{\boldsymbol{X}} < 1.4)$. [3]

(iii) Justify the use of the normal distribution in part **(ii)**. [1]

6 Stephan is an athlete who competes in the high jump. In the past, Stephan has succeeded in 90% of jumps at a certain height. He suspects that his standard has recently fallen and he decides to carry out a hypothesis test to find out whether he is right. If he succeeds in fewer than 17 of his next 20 jumps at this height, he will conclude that his standard has fallen.

(i) Find the probability of a Type I error. [4]

(ii) In fact Stephan succeeds in 18 of his next 20 jumps. Which of the errors, Type I or Type II, is possible? Explain your answer. [2]

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7 A random variable $\boldsymbol{X}$ has probability density function given by

$$f(\boldsymbol{x}) = \begin{cases} \dfrac{\boldsymbol{k}}{\boldsymbol{x}} & 1 \leqslant \boldsymbol{x} \leqslant \boldsymbol{a}, \\ 0 & \text{otherwise,} \end{cases}$$

where $\boldsymbol{k}$ and $\boldsymbol{a}$ are positive constants.

(i) Show that $\boldsymbol{k} = \dfrac{1}{\ln \boldsymbol{a}}$. [3]

(ii) Find $E(\boldsymbol{X})$ in terms of $\boldsymbol{a}$. [3]

(iii) Find the median of $\boldsymbol{X}$ in terms of $\boldsymbol{a}$. [4]

8 **(i)** The following tables show the probability distributions for the random variables $\boldsymbol{V}$ and $\boldsymbol{W}$.

$\boldsymbol{v}$	-1	0	1	>1
$P(\boldsymbol{V} = \boldsymbol{v})$	0.368	0.368	0.184	0.080

$\boldsymbol{w}$	0	0.5	1	>1
$P(\boldsymbol{W} = \boldsymbol{w})$	0.368	0.368	0.184	0.080

For each of the variables $\boldsymbol{V}$ and $\boldsymbol{W}$ state how you can tell from its probability distribution that it does NOT have a Poisson distribution. [2]

(ii) The random variable $\boldsymbol{X}$ has the distribution $\text{Po}(\lambda)$. It is given that

$$P(\boldsymbol{X} = 0) = \boldsymbol{p} \quad \text{and} \quad P(\boldsymbol{X} = 1) = 2.5\boldsymbol{p},$$

where $\boldsymbol{p}$ is a constant.

(a) Show that $\lambda = 2.5$. [1]

(b) Find $P(\boldsymbol{X} \geqslant 3)$. [2]

(iii) The random variable $\boldsymbol{Y}$ has the distribution $\text{Po}(\mu)$, where $\mu > 30$. Using a suitable approximating distribution, it is found that $P(\boldsymbol{Y} > 40) = 0.5793$ correct to 4 decimal places. Find μ. [5]

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