UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/31**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

* 4 0 6 4 0 1 4 4 9 0 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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1 Find the quotient and remainder when $2x^2$ is divided by $x + 2$. [3]

2 Expand $\dfrac{1 + 3x}{\sqrt{(1 + 2x)}}$ in ascending powers of x up to and including the term in x^2, simplifying the coefficients. [4]

3 Express $\dfrac{7x^2 - 3x + 2}{x(x^2 + 1)}$ in partial fractions. [5]

4 **(i)** Solve the equation $|4x - 1| = |x - 3|$. [3]

(ii) Hence solve the equation $\left|4^{y+1} - 1\right| = |4^y - 3|$ correct to 3 significant figures. [3]

5 For each of the following curves, find the gradient at the point where the curve crosses the y-axis:

(i) $y = \dfrac{1 + x^2}{1 + \mathrm{e}^{2x}}$; [3]

(ii) $2x^3 + 5xy + y^3 = 8$. [4]

6 The points P and Q have position vectors, relative to the origin O, given by

$$\overrightarrow{OP} = 7\mathbf{i} + 7\mathbf{j} - 5\mathbf{k} \quad \text{and} \quad \overrightarrow{OQ} = -5\mathbf{i} + \mathbf{j} + \mathbf{k}.$$

The mid-point of PQ is the point A. The plane Π is perpendicular to the line PQ and passes through A.

(i) Find the equation of Π, giving your answer in the form $ax + by + cz = d$. [4]

(ii) The straight line through P parallel to the x-axis meets Π at the point B. Find the distance AB, correct to 3 significant figures. [5]

7 **(a)** Without using a calculator, solve the equation

$$3w + 2\mathrm{i}w^* = 17 + 8\mathrm{i},$$

where w^* denotes the complex conjugate of w. Give your answer in the form $a + b\mathrm{i}$. [4]

(b) In an Argand diagram, the loci

$$\arg(z - 2\mathrm{i}) = \tfrac{1}{6}\pi \quad \text{and} \quad |z - 3| = |z - 3\mathrm{i}|$$

intersect at the point P. Express the complex number represented by P in the form $r\mathrm{e}^{\mathrm{i}\theta}$, giving the exact value of θ and the value of r correct to 3 significant figures. [5]

8 **(a)** Show that $\displaystyle\int_2^4 4x \ln x \, \mathrm{d}x = 56 \ln 2 - 12$. [5]

(b) Use the substitution $u = \sin 4x$ to find the exact value of $\displaystyle\int_0^{\frac{1}{24}\pi} \cos^3 4x \, \mathrm{d}x$. [5]

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9 **(i)** Express $4\cos\theta + 3\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the value of α correct to 4 decimal places. [3]

(ii) Hence

(a) solve the equation $4\cos\theta + 3\sin\theta = 2$ for $0 < \theta < 2\pi$, [4]

(b) find $\displaystyle\int \frac{50}{(4\cos\theta + 3\sin\theta)^2}\,\mathrm{d}\theta$. [3]

10 Liquid is flowing into a small tank which has a leak. Initially the tank is empty and, t minutes later, the volume of liquid in the tank is $V\,\mathrm{cm}^3$. The liquid is flowing into the tank at a constant rate of $80\,\mathrm{cm}^3$ per minute. Because of the leak, liquid is being lost from the tank at a rate which, at any instant, is equal to $kV\,\mathrm{cm}^3$ per minute where k is a positive constant.

(i) Write down a differential equation describing this situation and solve it to show that

$$V = \frac{1}{k}(80 - 80\mathrm{e}^{-kt}).$$ [7]

(ii) It is observed that $V = 500$ when $t = 15$, so that k satisfies the equation

$$k = \frac{4 - 4\mathrm{e}^{-15k}}{25}.$$

Use an iterative formula, based on this equation, to find the value of k correct to 2 significant figures. Use an initial value of $k = 0.1$ and show the result of each iteration to 4 significant figures. [3]

(iii) Determine how much liquid there is in the tank 20 minutes after the liquid started flowing, and state what happens to the volume of liquid in the tank after a long time. [2]

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