UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

MATHEMATICS **9709/33**

Paper 3 Pure Mathematics 3 **(P3)** **May/June 2012**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

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1 Expand $\frac{1}{\sqrt{(4+3x)}}$ in ascending powers of x, up to and including the term in x^2, simplifying the coefficients. [4]

2 Solve the equation $\ln(2x+3) = 2\ln x + \ln 3$, giving your answer correct to 3 significant figures. [4]

3 The parametric equations of a curve are

$$x = \sin 2\theta - \theta, \quad y = \cos 2\theta + 2\sin\theta.$$

Show that $\frac{dy}{dx} = \frac{2\cos\theta}{1+2\sin\theta}$. [5]

4 The curve with equation $y = \frac{e^{2x}}{x^3}$ has one stationary point.

(i) Find the x-coordinate of this point. [4]

(ii) Determine whether this point is a maximum or a minimum point. [2]

5 In a certain chemical process a substance A reacts with another substance B. The masses in grams of A and B present at time t seconds after the start of the process are x and y respectively. It is given that $\frac{dy}{dt} = -0.6xy$ and $x = 5e^{-3t}$. When $t = 0$, $y = 70$.

(i) Form a differential equation in y and t. Solve this differential equation and obtain an expression for y in terms of t. [6]

(ii) The percentage of the initial mass of B remaining at time t is denoted by p. Find the exact value approached by p as t becomes large. [2]

6 It is given that $\tan 3x = k\tan x$, where k is a constant and $\tan x \neq 0$.

(i) By first expanding $\tan(2x+x)$, show that

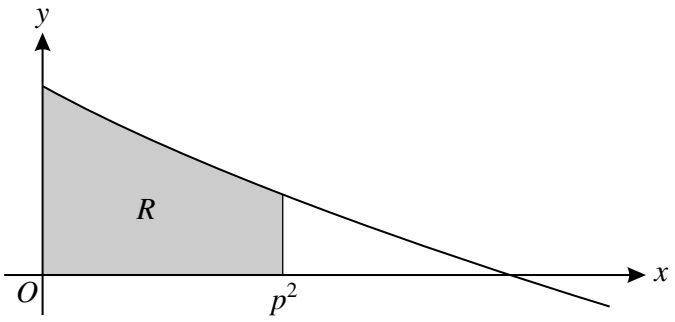
$$(3k-1)\tan^2 x = k - 3.$$ [4]

(ii) Hence solve the equation $\tan 3x = k\tan x$ when $k = 4$, giving all solutions in the interval $0^\circ < x < 180^\circ$. [3]

(iii) Show that the equation $\tan 3x = k\tan x$ has no root in the interval $0^\circ < x < 180^\circ$ when $k = 2$. [1]

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The diagram shows part of the curve $y = \cos(\sqrt{x})$ for $x \geqslant 0$, where x is in radians. The shaded region between the curve, the axes and the line $x = p^2$, where $p > 0$, is denoted by R. The area of R is equal to 1.

(i) Use the substitution $x = u^2$ to find $\int_0^{p^2} \cos(\sqrt{x})\,\mathrm{d}x$. Hence show that $\sin p = \dfrac{3 - 2\cos p}{2p}$. [6]

(ii) Use the iterative formula $p_{n+1} = \sin^{-1}\left(\dfrac{3 - 2\cos p_n}{2p_n}\right)$, with initial value $p_1 = 1$, to find the value of p correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

8 Let $\mathrm{f}(x) = \dfrac{4x^2 - 7x - 1}{(x+1)(2x-3)}$.

(i) Express $\mathrm{f}(x)$ in partial fractions. [5]

(ii) Show that $\int_2^6 \mathrm{f}(x)\,\mathrm{d}x = 8 - \ln\left(\frac{49}{3}\right)$. [5]

9 The lines l and m have equations $\mathbf{r} = 3\mathbf{i} - 2\mathbf{j} + \mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{j} + \mathbf{k})$ and $\mathbf{r} = 4\mathbf{i} + 4\mathbf{j} + 2\mathbf{k} + \mu(a\mathbf{i} + b\mathbf{j} - \mathbf{k})$ respectively, where a and b are constants.

(i) Given that l and m intersect, show that

$$2a - b = 4.$$ [4]

(ii) Given also that l and m are perpendicular, find the values of a and b. [4]

(iii) When a and b have these values, find the position vector of the point of intersection of l and m. [2]

10 (a) The complex numbers u and w satisfy the equations

$$u - w = 4\mathrm{i} \quad \text{and} \quad uw = 5.$$

Solve the equations for u and w, giving all answers in the form $x + \mathrm{i}y$, where x and y are real. [5]

(b) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers satisfying the inequalities $|z - 2 + 2\mathrm{i}| \leqslant 2$, $\arg z \leqslant -\frac{1}{4}\pi$ and $\mathrm{Re}\, z \geqslant 1$, where $\mathrm{Re}\, z$ denotes the real part of z. [5]

(ii) Calculate the greatest possible value of $\mathrm{Re}\, z$ for points lying in the shaded region. [1]

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