UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/12**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

* 4 4 5 7 1 1 4 7 8 6 *

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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1 Find $\int \left(x^3 + \frac{1}{x^3}\right) \mathrm{d}x$. [3]

2 **(i)** Find the terms in x^2 and x^3 in the expansion of $\left(1 - \frac{3}{2}x\right)^6$. [3]

(ii) Given that there is no term in x^3 in the expansion of $(k + 2x)\left(1 - \frac{3}{2}x\right)^6$, find the value of the constant k. [2]

3 The equation $x^2 + px + q = 0$, where p and q are constants, has roots -3 and 5.

(i) Find the values of p and q. [2]

(ii) Using these values of p and q, find the value of the constant r for which the equation $x^2 + px + q + r = 0$ has equal roots. [3]

4 A curve has equation $y = \frac{4}{3x - 4}$ and $P\,(2,\ 2)$ is a point on the curve.

(i) Find the equation of the tangent to the curve at P. [4]

(ii) Find the angle that this tangent makes with the x-axis. [2]

5 **(i)** Prove the identity $\frac{\cos\theta}{\tan\theta(1 - \sin\theta)} \equiv 1 + \frac{1}{\sin\theta}$. [3]

(ii) Hence solve the equation $\frac{\cos\theta}{\tan\theta(1 - \sin\theta)} = 4$, for $0^\circ \leqslant \theta \leqslant 360^\circ$. [3]

6 The function f is defined by $\mathrm{f} : x \mapsto \frac{x + 3}{2x - 1},\ x \in \mathbb{R},\ x \neq \frac{1}{2}$.

(i) Show that $\mathrm{ff}(x) = x$. [3]

(ii) Hence, or otherwise, obtain an expression for $\mathrm{f}^{-1}(x)$. [2]

7 The line L_1 passes through the points $A\,(2,\ 5)$ and $B\,(10,\ 9)$. The line L_2 is parallel to L_1 and passes through the origin. The point C lies on L_2 such that AC is perpendicular to L_2. Find

(i) the coordinates of C, [5]

(ii) the distance AC. [2]

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8 Relative to the origin O, the position vectors of the points A, B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 10 \\ 0 \\ 6 \end{pmatrix}.$$

(i) Find angle ABC. [6]

The point D is such that $ABCD$ is a parallelogram.

(ii) Find the position vector of D. [2]

9 The function f is such that $\mathrm{f}(x) = 3 - 4\cos^k x$, for $0 \leqslant x \leqslant \pi$, where k is a constant.

(i) In the case where $k = 2$,

(a) find the range of f, [2]

(b) find the exact solutions of the equation $\mathrm{f}(x) = 1$. [3]

(ii) In the case where $k = 1$,

(a) sketch the graph of $y = \mathrm{f}(x)$, [2]

(b) state, with a reason, whether f has an inverse. [1]

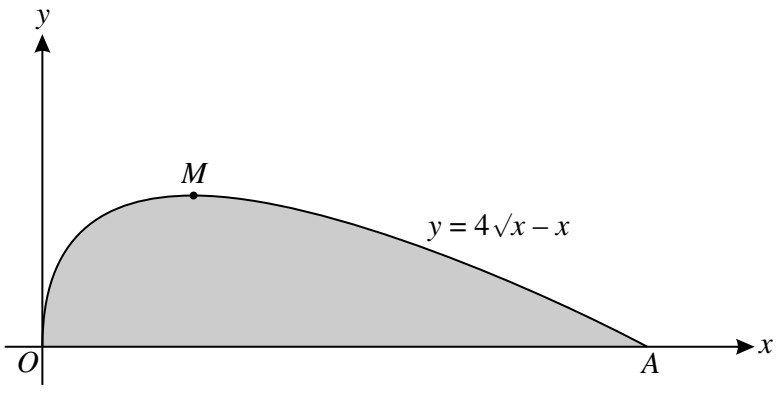
10 **(a)** A circle is divided into 6 sectors in such a way that the angles of the sectors are in arithmetic progression. The angle of the largest sector is 4 times the angle of the smallest sector. Given that the radius of the circle is 5 cm, find the perimeter of the smallest sector. [6]

(b) The first, second and third terms of a geometric progression are $2k + 3$, $k + 6$ and k, respectively. Given that all the terms of the geometric progression are positive, calculate

(i) the value of the constant k, [3]

(ii) the sum to infinity of the progression. [2]

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The diagram shows part of the curve $y = 4\sqrt{x} - x$. The curve has a maximum point at M and meets the x-axis at O and A.

(i) Find the coordinates of A and M. [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x-axis, giving your answer in terms of π. [6]

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