UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/21 Paper 2, maximum raw mark 50

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only - often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1 A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA -1 This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

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1 *EITHER* Attempt to square both sides obtaining three terms on each side M1
Attempt solution of three-term quadratic equation M1
Obtain $5x + 4x - 9 = 0$ and hence $-\frac{9}{5}$ and 1 A1

OR Obtain value 1 from graphical method, inspection or linear equation B1
Obtain value $-\frac{9}{5}$ similarly B2 [3]

2 State $\frac{\mathrm{d}x}{\mathrm{d}t} = 3 + 2\cos 2t$ or $\frac{\mathrm{d}y}{\mathrm{d}t} = -4\sin 2t$ (or both) B1
Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t} \div \frac{\mathrm{d}x}{\mathrm{d}t}$ M1
Obtain or imply $\frac{-4\sin 2t}{3 + 2\cos 2t}$ A1
Substitute $\frac{1}{6}\pi$ to obtain $-\frac{1}{2}\sqrt{3}$ or exact equivalent A1 [4]

3 State or imply that $\ln y = \ln K + m \ln x$ B1
Equate intercept on axis for $\ln y$ to $\ln K$ M1
Obtain 7.39 for K A1
Attempt calculation of gradient of line M1
Obtain 1.37 for m A1 [5]

4 **(i)** Substitute –2 and equate to zero or divide by $x + 2$ and equate remainder to zero M1
Obtain $a = 8$ A1 [2]

(ii) Attempt to find quotient by division or inspection or use of identity M1
Obtain at least $3x^2 + 2x$ A1
Obtain $3x^2 + 2x + 4$ with no errors seen A1 [3]

5 **(i)** Differentiate $\ln(x - 3)$ to obtain $\frac{1}{x-3}$ B1
Attempt to use product rule M1
Obtain $\ln(x-3) + \frac{x}{x-3}$ or equivalent A1
Substitute 4 to obtain 4 A1 [4]

(ii) Use correct quotient or product rule M1
Obtain correct derivative in any form, e.g. $\frac{(x+1)-(x-1)}{(x+1)^2}$ A1
Substitute 4 to obtain $\frac{2}{25}$ A1 [3]

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6 **(a)** Rewrite integrand as $12e^{x} + 4e^{3x}$ B1
Integrate to obtain $12e^{x}$ … B1
Integrate to obtain … $+\frac{4}{3}e^{3x}$ B1
Include … + c B1 [4]

(b) Use identity $\tan^2\theta = \sec^2\theta - 1$ B1
Integrate to obtain $2\tan\theta + \theta$ or equivalent B1
Use limits correctly for integral of form $a\tan\theta + b\theta$ M1
Confirm given answer $\frac{1}{2}(8+\pi)$ A1 [4]

7 **(i)** Draw correct sketch of $y = e^{2x}$ B1
Draw correct sketch of $y = 14 - x^2$ B1
Indicate two real roots only from correct sketches B1 [3]

(ii) Consider sign of $e^{2x} + x^2 - 14$ for 1.2 and 1.3 or equivalent M1
Justify conclusion with correct calculations (f(1.2) = –1.54, f(1.3) = 1.15) A1 [2]

(iii) Confirm given answer $x = \frac{1}{2}\ln(14 - x^2)$ B1 [1]

(iv) Use the iteration process correctly at least once M1
Obtain final answer 1.26 A1
Show sufficient iterations to 4 decimal places to justify answer or show a sign change in the interval (1.255, 1.256) A1 [3]
[1.2 → 1.2653 → 1.2588 → 1.2595 ;
1.25 → 1.2604 → 1.2593 → 1.2594 ;
1.3 → 1.2522 → 1.2598 → 1.2594]

8 **(i)** State or imply $R = \sqrt{52}$ or $2\sqrt{13}$ B1
Use appropriate formula to find α M1
Obtain 56.31° A1 [3]

(ii) Attempt to find at least one value of θ – α M1
Obtain one correct value 80.9° of θ A1
Carry out correct method to find second answer M1
Obtain 211.7° and no others in range A1 [4]

(iii) Obtain 60, following their value of R B1 √
Obtain 8. Allow quoted solution B1 [2]

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