UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the May/June 2011 question paper

for the guidance of teachers

9709 MATHEMATICS

9709/12 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- Cambridge will not enter into discussions or correspondence in connection with these mark schemes.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF Any Equivalent Form (of answer is equally acceptable)

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

BOD Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

MR Misread

PA Premature Approximation (resulting in basically correct work that is insufficiently accurate)

SOS See Other Solution (the candidate makes a better attempt at the same question)

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1	$\int\left(x^3+\frac{1}{x^3}\right)\mathrm{d}x = \frac{x^4}{4}+\frac{x^{-2}}{-2}+c$	3 × B1 [3]	Allow unsimplified, 1 mark for each term, including "c"
2	$\left(1-\frac{3}{2}x\right)^6$		
	(i) Term in x^2 $\quad {}^6C_2\times\left(\frac{\pm 3x}{2}\right)^2=\frac{135x^2}{4}$	M1 A1	For either unsimplified term co
	Term in x^3 $\quad {}^6C_3\times\left(\frac{\pm 3x}{2}\right)^3=\frac{-540x^3}{8}$	A1 [3]	co (omission or error with "–" can still gain 2 out of 3)
	(ii) Term in $x^3 = \frac{270x^3}{4}-\frac{135kx^3}{2}$	M1	considers exactly 2 terms in x^3
	$\rightarrow \quad k=1.$	A1 [2]	co
3	**(i)** $x^2+px+q = (x+3)(x-5)$	M1	Must be $(x+3)$ and $(x-5)$.
	$\rightarrow \quad p=-2,\ q=-15.$ (any other method ok)	A1 [2]	co
	(ii) $x^2+px+q+r=0$		
	Use of "b^2-4ac" Uses a, b and c correctly $r=16$	M1 DM1 A1 [3]	Any use of "b^2-4ac" c must include both q and r. co
	or $=(x+k)^2 \rightarrow 2k=p$ (M1) $\quad k^2=q+r$ (M1) $\rightarrow \quad k=-1 \quad \rightarrow \quad r=16$ (A1)		
4	$y=\frac{4}{3x-4}$		
	(i) $\frac{\mathrm{d}y}{\mathrm{d}x} = -4(3x-4)^{-2} \quad \times \quad 3$	B1 B1	Correct without ×3. For ×3.
	If $x=2$, $m=-3$ Eqn of tangent $\quad y-2=-3(x-2)$	M1 A1 [4]	Correct line eqn. co (for normal M0A0)
	(ii) $\tan\theta=\pm(-3)$ $\rightarrow \quad \theta=\pm 108.4^\circ$ (or $\pm 71.6^\circ$)	M1 A1√	Correct link with (± his gradient) co (accept acute or obtuse) or -71.6° or radians
	or scalar product, $\tan\theta$ = y-step ÷ x-step **or** use of tan $(A-B)$ M1A1 for each	[2]	

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5	**(i)** $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} \equiv \dfrac{\cos^2\theta}{\sin\theta(1-\sin\theta)}$	M1	Use of t = s ÷ c
	$= \dfrac{1-\sin^2\theta}{\sin\theta(1-\sin\theta)}$	M1	Replaces $\cos^2\theta$ with $1-\sin^2\theta$ to form f(sinθ).
	$= \dfrac{1+\sin\theta}{\sin\theta} = \dfrac{1}{\sin\theta}+1$	A1 [3]	AG. Ensure all ok. Must show difference of 2 squares.
	(ii) $\dfrac{\cos\theta}{\tan\theta(1-\sin\theta)} = 4 \rightarrow \dfrac{1}{\sin\theta}+1=4$	M1	Linking up to obtain $\sin\theta = k$.
	$\rightarrow \sin\theta = ⅓ \rightarrow \theta = 19.5^\circ, 160.5^\circ$	A1 A1√ [3]	co. √ $180^\circ - 1^{st}$ answer providing there are no other solutions in the range 0º to 360º.
6	**(i)** $f(x) = \dfrac{x+3}{2x-1}$		
	$ff(x) = \dfrac{\dfrac{x+3}{2x-1}+3}{\dfrac{2(x+3)}{2x-1}-1} = \dfrac{7x}{7} = x$	B1 M1 A1 [3]	Replacing "x" twice - must be correct Correct algebra – clearing $(2x-1)$ AG – all correct.
	(ii) $y = \dfrac{x+3}{2x-1}$		
	$\rightarrow \quad 2xy - y = x + 3$ $\rightarrow \quad x(2y-1) = y+3$	M1	Attempt to make x the subject and complete method
	$\rightarrow \quad f^{-1}(x) = \dfrac{x+3}{2x-1}$	A1 [2]	co
	or since $ff(x) = x$, $f^{-1}(x) = f(x) = \dfrac{x+3}{2x-1}$ (M1, A1)		
7	**(i)** (2, 5) to (10, 9) gradient = ½	B1	co
	Equation of L_2 $y = \frac{1}{2}x$.	B1√	√ on gradient of L_1
	Gradient of perpendicular = − 2	M1	Use of $m_1m_2 = -1$
	Eqn of Perp $y - 5 = -2(x-2)$	M1	Correct form of line eqn
	Sim Eqns $\rightarrow$ $C(3.6, 1.8)$	A1 [5]	co
	(ii) $d^2 = 1.6^2 + 3.2^2 \rightarrow d = 3.58$	M1 A1 [2]	Correct method for AC co (accept with √5 in answer)

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8	**(i)** $\overrightarrow{BA}.\overrightarrow{BC}$ or $\overrightarrow{AB}.\overrightarrow{CB}$	B1	Correct two vectors for angle *ABC*.
	$\overrightarrow{BA} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$	M1	Correct method for one of the sides.
	$\overrightarrow{BA}.\overrightarrow{BC} = -8$	M1	Correct use for **any** pair of vectors.
	$= 3 \times 7 \times \cos\theta$	M1	Correct method for moduli.
	$\rightarrow \quad \theta = 112.4^\circ$ or 1.96 radians	M1 A1 [6]	All linked correctly. co (67.6º usually gets 4/6)
	(ii) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC}$	M1	Correct method. (allow for **d** = **a** + **b** – **c** or for **d** = **a** + **c** – **b** or for **d** = **b** + **c** – **a**)
	$= \begin{pmatrix} 8 \\ 1 \\ 8 \end{pmatrix}$	A1√ [2]	A1√ for his $\overrightarrow{BC}$.
9	**(i)** **(a)** $f(x) = 3 - 4\cos^2 x$. One limit is -1	B1	co irrespective of inequalities
	Other limit is 3	B1 [2]	co irrespective of inequalities
	(b) $3 - 4\cos^2 x = 1 \rightarrow \cos^2 x = \frac{1}{2}$		
	$\rightarrow \quad \cos x = \pm\frac{1}{\sqrt{2}}$	M1	Makes cos *x* the subject.
	$\rightarrow \quad x = \frac{1}{4}\pi$ or $\frac{3}{4}\pi$	A1 A1√ [3]	co (radians). √ for "π − (1st answer)" ("exact" means that decimal answers only earn A0 A1√)
	(ii) **(a)**	B1	Joins (0, −1) to (π, 7), providing increasing function
		B1 [2]	Not a line, flattens at extremities-needs inflexion.
	(b) f has an inverse since it is 1:1 or increasing or no turning points.	B1 [1]	co independent of part (**i**)

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10	**(a)** $a + 5d = 4a$ or $\frac{(a+4a)}{2} \times 6$	B1	co
	$\frac{6}{2}(2a + 5d)$ or $\frac{(a+4a)}{2} \times 6 = 360$	M1 A1	Correct left-hand side. All correct.
	Sim Eqns $a = 24^\circ$ or $\frac{2\pi}{15}$ rads	A1	Either answer.
	Arc length = 5θ Perimeter = 12.1.	M1 A1 [6]	Correct use of arc length with θ in rads. co
	(b) **(i)** $\frac{k+6}{2k+3} = \frac{k}{k+6}$	M1 A1	Correct eqn for k.
	$\rightarrow\ k^2 - 9k - 36 = 0\ \rightarrow\ k = 12$ (NB stating a, ar, ar^2 as f(k) gets M1)	A1 [3]	Co condone inclusion of $k = -3$.
	(ii) $r = ⅔,\quad a = 27$ $\rightarrow\ S_\infty = 27 \div ⅓ = 81.$	M1 A1 [2]	Correct formula for S_∞ must have $-1 \le r \le 1$. co.
11	$y = 4\sqrt{x} - x.$		
	(i) At A, $4\sqrt{x} - x = 0\ \rightarrow\ A(16, 0)$	B1	co – independent of working.
	$\frac{dy}{dx} = 2x^{-\frac{1}{2}} - 1$	B1 B1	B1 for each part.
	$= 0$ when $x = 4\ \rightarrow\ (4, 4)$	M1 A1 [5]	Sets to 0 and solves his eqn. co
	(ii) Vol = $\pi \int y^2\ dx =$		
	$\pi \int (16x + x^2 - 8x^{\frac{3}{2}})\ dx$	M1	Use of correct formula + attempt at integration
	$\pi[8x^2 + \frac{x^3}{3} - 8\frac{x^{\frac{5}{2}}}{\frac{5}{2}}]$	A3,2,1	One mark for each term – unsimplified
	Limits 0 to 16 $\rightarrow$ 136.5π. (or 137π)	DM1 A1 [6]	Correct use of his limits. co – (429 ok)

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