Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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8017897469

MATHEMATICS **9709/22**

Paper 2 Pure Mathematics 2 **February/March 2023**

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 Find the exact value of $\int_0^{\frac{1}{2}\pi} 2\tan^2\left(\frac{1}{2}x\right) \mathrm{d}x$. [4]

2 Solve the equation $\tan(\theta - 60°) = 3\cot\theta$ for $-90° < \theta < 90°$. [5]

3 The polynomial $\mathrm{p}(x)$ is defined by

$$\mathrm{p}(x) = ax^3 - ax^2 + ax + b,$$

where a and b are constants. It is given that $(x + 2)$ is a factor of $\mathrm{p}(x)$, and that the remainder is 35 when $\mathrm{p}(x)$ is divided by $(x - 3)$.

(a) Find the values of a and b. [5]

(b) Hence factorise $p(x)$ and show that the equation $p(x) = 0$ has exactly one real root. [3]

4 **(a)** Sketch, on the same diagram, the graphs of $y = |2x - 11|$ and $y = 3x - 3$. [2]

(b) Solve the inequality $|2x - 11| < 3x - 3$. [3]

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(c) Find the smallest integer N satisfying the inequality $|2\ln N - 11| < 3\ln N - 3$. [2]

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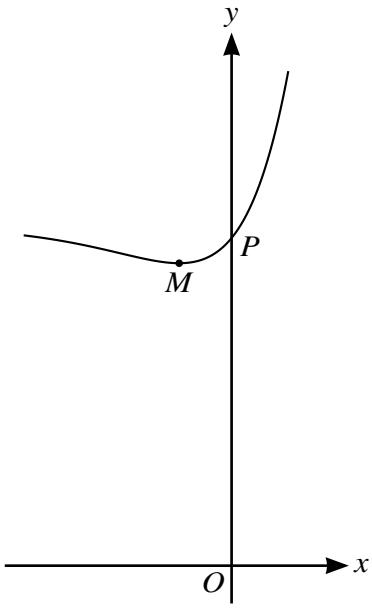
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5 It is given that $\int_1^a \left(\frac{4}{1+2x} + \frac{3}{x} \right) \mathrm{d}x = \ln 10$, where a is a constant greater than 1.

(a) Show that $a = \sqrt[3]{90(1+2a)^{-2}}$. [5]

(b) Use an iterative formula, based on the equation in **(a)**, to find the value of a correct to 3 significant figures. Use an initial value of 1.7 and give the result of each iteration to 5 significant figures. [3]

6



The diagram shows the curve with equation $y = \dfrac{4e^{2x} + 9}{e^{x} + 2}$. The curve has a minimum point M and crosses the y-axis at the point P.

(a) Find the exact value of the gradient of the curve at P. [4]

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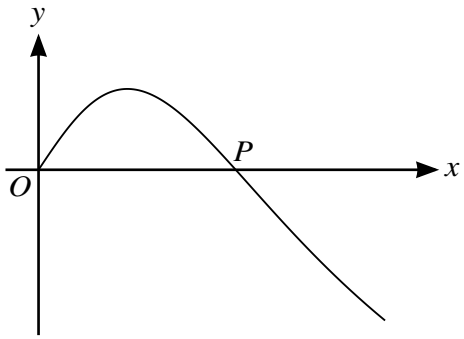
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(b) Find the exact coordinates of M. [4]

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The diagram shows the curve with parametric equations

$$x = k\tan t, \qquad y = 3\sin 2t - 4\sin t,$$

for $0 < t < \frac{1}{2}\pi$. It is given that k is a positive constant. The curve crosses the x-axis at the point P.

(a) Find the value of $\cos t$ at P, giving your answer as an exact fraction. [3]

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(b) Express $\frac{dy}{dx}$ in terms of k and $\cos t$. [4]

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(c) Given that the normal to the curve at P has gradient $\frac{9}{10}$, find the value of k, giving your answer as an exact fraction. [3]

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Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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