Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability & Statistics 1 **February/March 2022**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the February/March 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	X: –2, –1, 0, 1, 2 P(X): $\frac{1}{16}$, $\frac{3}{16}$, $\frac{5}{16}$, $\frac{5}{16}$, $\frac{2}{16}$ 0.0625, 0.1875, 0.3125, 0.3125, 0.125	**B1**	Table with correct X values and at least one probability $0 < p < 1$. Condone any additional X values if probability stated as 0. No repeated X values.
		B1	3 correct probabilities linked with correct outcomes, may not be in table.
		B1	2 further correct probabilities linked with correct outcomes, may not be in table No repeated X values. **SC** if less than 3 correct probabilities seen, award **SCB1** Sum of *their* probabilities, $0 < p < 1$, of 4,5 or 6 X values = 1 (condone summing to 1±0·01 or better).
		3	
1(b)	$\left[\frac{1}{16}\times -2^2 + \frac{3}{16}\times -1^2 \left(+\frac{5}{16}\times 0^2\right) + \frac{5}{16}\times 1^2 + \frac{2}{16}\times 2^2 - \left(\frac{1}{4}\right)^2\right]$ $\frac{1\times 4 + 3\times 1 + 5\times 0 + 5\times 1 + 2\times 4}{16} - 0.25^2$	**M1**	Appropriate variance formula using $(E(X))^2$ value, accept unsimplified. FT *their* table with at least 3 different X values even if probabilities not summing to 1, $0 < p < 1$. Condone 1 error providing all probabilities <1 and 0.25^2 used
	$\left[= \frac{5}{4} - \frac{1}{16} = \right] \frac{19}{16}$, 1.1875	**A1**	Condone 1.188 or 1.19 WWW
		2	

Question	Answer	Marks	Guidance
2(a)	[P(>2) = 1 – P(0,1,2) =] $1-({}^{7}C_{0}\ 0.18^{0}\ 0.82^{7}\ +{}^{7}C_{1}\ 0.18^{1}\ 0.82^{6}\ +{}^{7}C_{2}\ 0.18^{2}\ 0.82^{5})$	**M1**	One term ${}^{7}C_{x}\ p^{x}(1-p)^{7-x}$, $0<p<1, 0<x<7$
	= 1 – (0.249285 + 0.383048 + 0.252251) = 1 – 0.88458	**A1**	Correct unsimplified expression or better Condone omission of brackets if recovered
	0.115	**B1**	WWW. $0{\cdot}115 \leqslant p < 0{\cdot}1155$ not from wrong working
		3	
2(b)	[P(at least 1 day of rain) = 1 – P(0) = $1-(0.82)^{7}$ =] 0.7507	**B1**	AWRT 0.751 seen
	[P(exactly 2 periods) =] $0.7507^{2}\times(1-0.7507)\times 3$	**M1**	FT *their* $1-p^{7}$ or *their* 0.7507 if identified, not 0.18, 0.82 Accept $\times{}^{3}C_{r}$, r=1,2 or $\times{}^{3}P_{1}$ for ×3 Condone ×2
	0.421	**A1**	Accept $0.421 \leqslant p \leqslant 0.4215$ **SC B1** if 0/3 scored for final answer only $0.421 \leqslant p \leqslant 0.4215$
		3	

<table>
<tr><th>Question</th><th>Answer</th><th>Marks</th><th>Guidance</th></tr>
<tr><td rowspan="5">3(a)</td><td rowspan="4"><table><tr><td>Class Width</td><td>30</td><td>15</td><td>20</td><td>10</td><td>25</td></tr><tr><td>Frequency Density</td><td>0.7</td><td>2</td><td>3.4</td><td>8.6</td><td>1.8</td></tr></table></td><td>M1</td><td>At least 4 frequency densities calculated</td></tr>
<tr><td>A1</td><td>All heights correct on graph</td></tr>
<tr><td>B1</td><td>Bar ends at 0·5, 30·5, 45·5, 65.5, 75.5, 100.5 (at axis), 5 bars drawn, condone 0 in first bar
0.5 ⩽ time axis ⩽ 100.5, linear scale with at least 3 values indicated.</td></tr>
<tr><td>B1</td><td>Axes labelled: Frequency density (fd), time (t) and mins (or appropriate title). Linear fd scale, with at least 3 values indicated 0 ⩽ fd axis ⩽ 8·6</td></tr>
<tr><td></td><td>4</td><td></td></tr>
<tr><td rowspan="2">3(b)</td><td>66 – 75</td><td>B1</td><td>Condone 65.5 – 75.5</td></tr>
<tr><td></td><td>1</td><td></td></tr>
<tr><td rowspan="2">3(c)</td><td>Distribution is not symmetrical</td><td>B1</td><td>Or skewed, ignore nature of skew</td></tr>
<tr><td></td><td>1</td><td></td></tr>
</table>

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Question	Answer	Marks	Guidance
4(a)	$\mathrm{P}(46 < X < 62) = \mathrm{P}\left(\frac{46-55}{6} < Z < \frac{62-55}{6}\right)$	**M1**	46 or 62, 55 and 6 substituted into ±standardisation formula once. Condone 6^2 and continuity correction ±0.5
	$= \mathrm{P}\left(-1.5 < Z < \frac{7}{6}\right)$	**B1**	Both standardisation values correct, accept unsimplified
	$\left[=\Phi\left(\frac{7}{6}\right)-\left(1-\Phi(1.5)\right)\right]$ $= 0.8784 + (0.9332 - 1)$	**M1**	Calculating the appropriate area from stated Φs of z-values, must be probabilities.
	0.812	**A1**	$0{\cdot}8115 < p \leqslant 0{\cdot}812$
		4	
4(b)	$z = \pm 0.674$	**B1**	CAO, critical z-value
	$\frac{36-42}{\sigma} = -0.674$	**M1**	36 and 42 substituted in ±standardisation formula, no continuity correction, not σ^2, $\surd\sigma$, equated to a z-value
	$\sigma = 8.9[0]$	**A1**	WWW. Only dependent on M.
		3	

Question	Answer	Marks	Guidance
4(c)	P(male < 46) = 1−*their* 0.9332 = 0.0668	**M1**	FT value from part **(a)** or Correct: $1-\Phi\left(\frac{46-55}{6}\right)$,condone continuity correction, σ^2, $\sqrt{\sigma}$, **and** probability found. Condone unsupported correct value stated.
	P(female < 46) = $P(Z < \frac{46-42}{their\,8.90}) = \Phi(0.449) = 0.6732$	**M1**	46, 42 and *their* 4(b) σ (or correct σ) substituted in ±standardisation formula, condone continuity correction, σ^2, $\sqrt{\sigma}$, **and** probability found Condone $\frac{4}{their\,8.90}$.
	P(both) = 0.0668 ×0.6732	**M1**	Product of *their* 2 probabilities (0 < both < 1) Not 0.25 or *their* final answer to **4(a)** used.
	0.0450 or 0.0449	**A1**	$0{\cdot}0449 \leqslant p \leqslant 0{\cdot}0450$
		4	

Question	Answer	Marks	Guidance
5(a)	$^5C_1 \times {}^7C_4$	**M1**	$^7C_4 \times k$, k integer $\geqslant 1$ Condone $^5P_1$ for M1 only
	175	**A1**	
		2	

Question	Answer	Marks	Guidance
5(b)	2B 1G 2A $^3C_2 \times {}^4C_1 \times {}^5C_2 = 120$ 2B 2G 1A $^3C_2 \times {}^4C_2 \times {}^5C_1 = 90$ 2B 3G $^3C_2 \times {}^4C_3 = 12$ 3B 1G 1A $^3C_3 \times {}^4C_1 \times {}^5C_1 = 20$ 3B 2G $^3C_3 \times {}^4C_2 = 6$	**M1**	$^3C_x \times {}^4C_y \times {}^5C_z$, $x + y + z = 5$, x,y,z integers $\geqslant 1$ Condone use of permutations for this mark
		B1	2 appropriate identified outcomes correct, allow unsimplified
		M1	Summing *their* values for 4 or 5 correct identified scenarios only (no repeats or additional scenarios), condone identification by unsimplified expressions
	[Total =] 248	**A1**	Note: Only dependent upon M marks
		4	
5(c)	$8! \times 3! \times {}^5P_2$	**M1**	$8! \times m$, m an integer $\geqslant 1$ Accept $8 \times 7!$ for $8!$
		M1	$3! \times n$, n an integer > 1
		M1	$p \times {}^5P_2$, $p \times {}^5C_2 \times 2$, $p \times 20$, p an integer > 1 If extra terms present, maximum 2/3 M marks available
	4 838 400	**A1**	Exact value required
		4	

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Question	Answer	Marks	Guidance
6(a)	[Probability of lemon = $\frac{3}{15}=\frac{1}{5}$] $\left[\left(\frac{4}{5}\right)^6\times\frac{1}{5}=\right]\frac{4096}{78125}$, 0.0524	**B1**	0.0524288 rounded to more than 3SF if final answer
		1	
6(b)	$\left(1-\frac{1}{5}\right)^6$	**M1**	or $\left(\frac{4}{5}\right)^6$. FT *their* $\frac{1}{5}$ or correct. From final answer Condone $\left(\frac{4}{5}\right)^5$ or $\left(\frac{1}{5}\right)\times\left(\frac{4}{5}\right)^5+\left(\frac{4}{5}\right)^6$
	$\frac{4096}{15625}$, 0.262	**A1**	0.262144 rounded to more than 3SF
	Alternative method for question 6(b)		
	[1 – P(1,2,3,4,5,[6]) =] $1-\left(\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}+\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}+\left(\frac{4}{5}\right)^5\times\frac{1}{5}\right)$	**M1**	From final answer Condone omission of $\left(\frac{4}{5}\right)^5\times\frac{1}{5}$
	$\frac{4096}{15625}$, 0.262	**A1**	0.262144 rounded to more than 3SF
		2	

Question	Answer	Marks	Guidance
6(c)	$\frac{10}{15}\times\frac{9}{14}\times\frac{8}{13}$	**M1**	$\frac{a}{15}\times\frac{a-1}{14}\times\frac{a-2}{13}$, no additional terms
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$\frac{3}{15}\times\frac{2}{14}\times\frac{1}{13}+3\times\frac{3}{15}\times\frac{2}{14}\times\frac{7}{13}+3\times\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}+\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}$	**M1**	[3Ls + 2Ls1S + 1L2Ss + 3Ss] Condone one numerator error. Condone no multiplications seen if tree diagram complete with probabilities on each branch, scenarios listed and attempt at evaluation
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$1-\left(\frac{5}{15}\times\frac{4}{14}\times\frac{3}{13}+3\times\frac{5}{15}\times\frac{4}{14}\times\frac{10}{13}+3\times\frac{5}{15}\times\frac{10}{14}\times\frac{9}{13}\right)$	**M1**	1 – P(3,2,1 oranges) Condone one numerator error.
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
	Alternative method for question 6(c)		
	$\frac{{}^{10}C_3}{{}^{15}c_3}$	**M1**	
	$\frac{24}{91}$, 0·264	**A1**	0.263736 rounded to more than 3SF
		2	

Question	Answer	Marks	Guidance
6(d)	$\frac{7}{15}\times\frac{5}{14}\times\frac{3}{13}\times 3!$	**M1**	All probabilities of the form: $\frac{7}{a}\times\frac{5}{b}\times\frac{3}{c}$, $13 \leqslant a,b,c \leqslant 15$
		M1	$\frac{e}{f}\times\frac{g}{h}\times\frac{i}{j}\times 3!$ e,f,g,h,i,j positive integers forming probabilities or 6 identical probability calculations or values added, no additional terms
	$\frac{3}{13}$, 0·231	**A1**	0·230769 rounded (not truncated) to more than 3SF
	Alternative method for question 6(d)		
	$\frac{{}^3\mathrm{C}_1\times{}^5\mathrm{C}_1\times{}^7\mathrm{C}_1}{{}^{15}\mathrm{C}_3}$	**M1**	$\frac{{}^3\mathrm{C}_1\times{}^5\mathrm{C}_1\times{}^7\mathrm{C}_1}{k}$, k integer > 1 Condone use of permutations
		M1	$\frac{{}^3\mathrm{C}_a\times{}^5\mathrm{C}_b\times{}^7\mathrm{C}_c}{{}^{15}\mathrm{C}_3}$, $0<a<3$, $0<b<5$, $0<c<7$, Condone use of permutations
	$\frac{3}{13}$, 0·231	**A1**	0·230769 rounded (not truncated) to more than 3SF
		3	

Question	Answer	Marks	Guidance
6(e)	$\dfrac{\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}+\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}\times 3}{their(c)}\left[=\frac{14}{65}\div\frac{24}{91}\right]$	**B1**	$\frac{3}{15}\times\frac{7}{14}\times\frac{6}{13}\times 3$ seen (SSL, SLS, LSS) **SC B1** $\frac{3}{65}\times 3, \frac{126}{2730}\times 3$ seen
		B1	$\frac{7}{15}\times\frac{6}{14}\times\frac{5}{13}$ seen in numerator (SSS) **SCB1** $\frac{210}{2730}, \frac{1}{13}$ seen in numerator
		M1	Fraction with *their* **(c)** or correct in denominator $\left(\frac{720}{2730}, \frac{24}{91}, 0.263736\right)$
	$=\frac{49}{60}$, 0·817	**A1**	Accept $0.81\dot{6}$
	Alternative method for question 6(e)		
	$\dfrac{{}^{7}C_{2}\times{}^{3}C_{1}+{}^{7}C_{3}}{{}^{10}C_{3}}$	**B1**	${}^{7}C_{2}\times{}^{3}C_{1}$ seen (SSL, SLS, LSS) **SCB1** 21 × 3 seen or use of permutations
		B1	${}^{7}C_{3}$ seen in numerator (SSS) **SCB1** 35 seen in numerator or use of permutations
		M1	Fraction with ${}^{10}C_{3}$ or consistent with *their* numerator of **6(c)** in denominator
	$=\frac{49}{60}$, 0·817	**A1**	Accept $0.81\dot{6}$
		4	

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