Cambridge Assessment International Education

Cambridge International AS & A Level

MATHEMATICS **9709/52**

Paper 5 Probability and Statistics 1 **March 2021**

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

DM or **DB** When a part of a question has two or more 'method' steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

FT Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
- For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
- The total number of marks available for each question is shown at the bottom of the Marks column.
- Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
- Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

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Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no 'follow through' from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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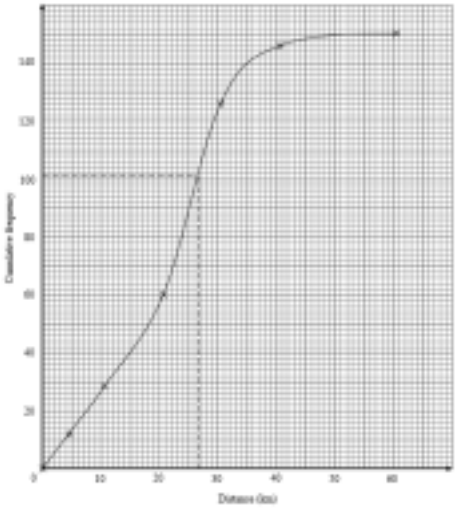
Question	Answer	Marks	Guidance
1(a)	$\left[\left(\frac{4}{5}\right)^7\frac{1}{5}=\right]\ \frac{16384}{390625}$ or 0·0419[43…]	**B1**	Evaluated, final answer.
		1	
1(b)	$1-\left(\frac{4}{5}\right)^5$ or $\frac{1}{5}+\frac{4}{5}\times\frac{1}{5}\left(\frac{4}{5}\right)^2\times\frac{1}{5}+\left(\frac{4}{5}\right)^3\times\frac{1}{5}+\left(\frac{4}{5}\right)^4\times\frac{1}{5}$	**M1**	$1-p^n$ $n = 5,6$ or $p + pq + pq^2+pq^3+ pq^4$ $(+ pq^5)$ $0 < p < 1, p + q = 1,$ Sum of a geometric series may be used.
	$\frac{2101}{3125}$ or 0·672[32]	**A1**	Final answer.
	Alternative method for question 1(b)		
	[P(at least 1 three scored in 5 throws) =] $\left(\frac{1}{5}\right)^5+{}^5\mathrm{C}_4\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)+{}^5\mathrm{C}_3\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2+{}^5\mathrm{C}_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3+{}^5\mathrm{C}_4\left(\frac{1}{5}\right)\left(\frac{4}{5}\right)^4$	**M1**	$(p)^5+{}^5\mathrm{C}_4(p)^4(q)+{}^5\mathrm{C}_3(p)^3(q)^2+{}^5\mathrm{C}_2(p)^2(q)^3+{}^5\mathrm{C}_1(p)(q)^4$ or $(p)^6+{}^6\mathrm{C}_5(p)^5(q)+{}^6\mathrm{C}_4(p)^4(q)^2+{}^6\mathrm{C}_3(p)^3(q)^3$ $+{}^6\mathrm{C}_2(p)^2(q)^4+{}^6\mathrm{C}_1(p)(q)^5,\ 0 < p < 1, p + q = 1$ At least first, last and one intermediate term is required to show pattern of terms if not all terms stated.
	$\frac{2101}{3125}$ or 0·672[32]	**A1**	Final answer.
		2	

Question	Answer	Marks	Guidance
2(a)	$0.2[\times 1]+0.45\times 0.4+0.35\times 0.3$	**M1**	0·2 [× 1] + 0·45 × *b* + 0·35 × *c*, *b* = 0·4, 0·6 *c* = 0·3, 0·7
	0·485 or $\frac{97}{200}$	**A1**	
		2	
2(b)	$P(Y \mid \overline{H}) = \frac{P(Y \cap \overline{H})}{P(\overline{H})} = \frac{0.35\times 0.7}{1-\textit{their}\textbf{(a)}} = \frac{0.245}{0.515}$	**B1**	0·35 × 0·7 or 0·245 seen as numerator or denominator of fraction.
		M1	0·515 or 1 – *their* **(a)** or [0·3 × 0 +] 0·45 × *d* + 0·35 × *e*, where *d = their b′, e = their c′* seen as denominator of fraction.
	0·476 or $\frac{49}{103}$	**A1**	0·4757 ⩽ *p* ⩽ 0·476
		3	

Question	Answer	Marks	Guidance
3(a)	$\mathrm{P}\left(\left(\frac{85-96}{18}\right) < z < \left(\frac{100-96}{18}\right)\right)$	**M1**	Use of ±standardisation formula once with appropriate values substituted, no continuity correction, not σ^2 or $\sqrt{\sigma}$.
	$\mathrm{P}(-0.6111 < z < 0.2222)$ $= \Phi(0.2222) + \Phi(0.6111) - 1$ $= 0.5879 + 0.7294 - 1$	**M1**	Appropriate area Φ, from final process, must be probability. Use of (1 – *z*) implies M0.
	0·317	**A1**	Final answer which rounds to 0·317.
		3	

Question	Answer	Marks	Guidance
3(b)	$z = \pm 1{\cdot}175$	**B1**	$1{\cdot}17 \leqslant z \leqslant 1{\cdot}18$ or $-1{\cdot}18 \leqslant z \leqslant -1{\cdot}17$
	$-1.175 = \dfrac{t-96}{18}$	**M1**	An equation using ±standardisation formula with a z-value, condone σ^2, $\sqrt{\sigma}$ or continuity correction. E.g. equating to 0·88, 0·12, 0·8106, 0·1894, 0·5478, 0·4522, ±0·175 or ±2·175 implies M0.
	74·85 or 74·9	**A1**	$74{\cdot}85 \leqslant t \leqslant 74{\cdot}9$
		3	

Question	Answer	Marks	Guidance
4(a)	x | 1 | 2 | 3 | 4 prob | $4k$ | $6k$ | $6k$ | $4k$	**B1**	Table with × values and one correct probability expressed in terms of k. Condone any additional × values if probability stated as 0.
		B1	Remaining 3 probabilities correct expressed in terms of k – condone if the first correct probability is not in table.
		2	
4(b)	$[4k + 6k + 6k + 4k = 1]\ k = \frac{1}{20}\ \ (= 0.05)$	**B1**	Correct value for k SOI. May be calculated in **4(a)**. **SC B1** If denominator $20k$ used throughout.
	$E(X) = 1\times\frac{4}{20} + 2\times\frac{6}{20} + 3\times\frac{6}{20} + 4\times\frac{4}{20} = \frac{4}{20} + \frac{12}{20} + \frac{18}{20} + \frac{16}{20}$ $(= 2.5)$	**M1**	Accept unsimplified expression. Condone $4k + 12k + 18k + 16k$. May be implied by use in Variance expression. **Special ruling:** Allow use of denominator $20k$.
	$\text{Var}(X) = 1^2\times\frac{4}{20} + 2^2\times\frac{6}{20} + 3^2\times\frac{6}{20} + 4^2\times\frac{4}{20} - \left(their\ 2\frac{1}{2}\right)^2$ $= (4 + 24 + 54 + 64)\times their\,0.05 - (their\,2.5)^2$ **Or** $(1-2.5)^2\times\frac{4}{20} + (2-2.5)^2\times\frac{6}{20} + (3-2.5)^2\times\frac{6}{20} + (4-2.5)^2\times\frac{4}{20}$	**M1**	Appropriate variance formula with *their* numerical probabilities using *their* $(\text{E}(X))^2$, accept unsimplified, with *their* k substituted. **Special ruling:** If denominator $20k$ used throughout, accept appropriate variance formula in terms of k.
	1·05	**A1**	AG, NFWW.
		4	

Question	Answer	Marks	Guidance
5(a)	<table><tr><td>Distance</td><td>0-4</td><td>5-10</td><td>11-20</td><td>21-30</td><td>31-40</td><td>41-60</td></tr><tr><td>Upper boundary</td><td>4·5</td><td>10·5</td><td>20·5</td><td>30·5</td><td>40·5</td><td>60·5</td></tr><tr><td>Cumulative frequency</td><td>12</td><td>28</td><td>60</td><td>126</td><td>146</td><td>150</td></tr></table>	**B1**	Correct cumulative frequencies seen (may be by table or plotted accurately on graph), condone 12 not stated.
		**B1**	Axes labelled 'distance (or d) [in] km' from 0 to 60 **and** 'cumulative frequency' (or cf) from 0 to 150.
		M1	At least 5 points plotted at upper end points for d (allow upper boundary ±0·5) with a linear scale for distance, condone 0 – 4 interval inaccurate, no scale break on axis. Not bar graph/histogram unless clear indication of upper end point only of each bar.
		A1	All plotted correctly at correct upper end points (4.5 etc.) with both scales linear ($0 \leqslant d \leqslant 60$, $0 \leqslant \text{cf} \leqslant 150$), curve drawn accurately joined to (0,0), cf line>150, no daylight if >150.
		4	
5(b)	70% of 150 = 105	**M1**	105 seen or implied by indication on grid.
	Approx. 27	**A1 FT**	Strict FT *their* increasing cumulative frequency graph, use of graph must be seen. If no clear evidence of use of graph: **SC B1 FT** correct value from *their* increasing cumulative frequency graph.
		2	

Question	Answer	Marks	Guidance
5(c)	Midpoints: 2.25, 7.5, 15.5, 25.5, 35.5, 50.5	**B1**	At least 5 correct midpoints seen.
	$\text{Mean} = \dfrac{2.25\times12 + 7.5\times16 + 15.5\times32 + 25.5\times66 + 35.5\times20 + 50.5\times4}{150}$ $= \dfrac{27+120+496+1683+710+202}{150}$	**M1**	Using 6 midpoint attempts (e.g. 2·25 ±0·5), condone one error not omission, multiplied by frequency, accept unevaluated, denominator either correct or *their* Σ frequencies.
	$\left[= \dfrac{3238}{150}\right] = 21.6,\ 21\dfrac{44}{75}$	**A1**	Evaluated, WWW, accept 21·5[866…].
		3	

Question	Answer	Marks	Guidance
6(a)	$\dfrac{11!}{2!2!2!}$	**M1**	11! alone as numerator. 2! × *m*! × *n*! on denominator, *m* = 1, 2, *n* = 1, 2. no additional terms, no additional operations.
	4989600	**A1**	Exact answer only.
		2	

Question	Answer	Marks	Guidance
6(b)	**Method 1** R ^ ^ ^ ^ ^ ^ ^ R		
	Arrange the 7 letters CTEPILL = $\frac{7!}{2!}$ Number of ways of placing As in non-adjacent places = ${}^8C_2$ $\frac{7!}{2!}\times {}^8C_2$	**B1**	$\frac{7!}{2!}\times k$ seen, k an integer > 1.
		M1	$m\times n(n-1)$ or $m\times {}^nC_2$ or $m\times {}^nP_2$, n = 7, 8 or 9, m an integer > 1.
		M1	$\frac{7!}{p!}\times {}^8C_2$ or $\frac{7!}{p!}\times {}^8P_2$, p integer $\geqslant$ 1, condone 2520×28.
	= 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
	Method 2 [Arrangements Rs at ends – Arrangements Rs at ends and As together]		
	Total arrangements with R at beg. and end = $\frac{9!}{2!2!}$ Arrangements with R at ends and As together = $\frac{8!}{2!}$ With As not together = $\frac{9!}{2!2!}-\frac{8!}{2!}$	**M1**	$\frac{9!}{2!m!}-k$, 90720 > k integer > 1, m = 1, 2.
		B1	$s-\frac{8!}{2!}$, s an integer >1
		M1	$\frac{9!}{p}-\frac{8!}{q}$, p, q integers $\geqslant$ 1, condone 90720 – 20160.
	[90720 – 20160] = 70560	**A1**	Exact answer only. **SC B1** 70560 from M0, M1 only.
		4	

Question	Answer	Marks	Guidance
6(c)	**Method 1**		
	R R A L _ _ $^5C_2$ = 10 R R A L L _ $^5C_1$ = 5 R R A A L _ $^5C_1$ = 5 R R A A L L = 1	**M1**	^5C_x seen alone or $^5C_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0 < x < 5$ linked to an appropriate scenario.
		A1	$^5C_2 \times k$, $k = 1$ oe or $^5C_1 \times m$, $m = 1,2$ oe alone. **SC** if ^5C_x not seen. **B2** for 5 or 10 linked to the appropriate scenario WWW.
		M1	Add outcomes from 3 or 4 identified correct scenarios only, accept unsimplified. $^2C_w \times {}^2C_x \times {}^2C_y \times {}^5C_z$, $w+x+y+z=6$ identifies w Rs, × As and y Ls.
	[Total =] 21	**A1**	WWW, only dependent on 2nd M mark. Note: $^5C_2 + {}^5C_1 + {}^5C_1 + 1 = 21$ is sufficient for 4/4.
			SC not all (or no) scenarios identified. **B1** 10 + 5 + 5 + 1 **DB1** = 21
	Method 2 – Fixing RRAL first. N.B. No other scenarios can be present anywhere in solution.		
	R R A L ^ ^ = $^7C_2$	**M1**	^7C_x seen alone or $^7C_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0<x<7$. Condone ^7P_x or $^7P_x \times k$, $2 \geqslant k \geqslant 1$, k an integer, $0<x<7$.
		M1	$^7C_2 \times k$, $2 \geqslant k \geqslant 1$oe
		A1	$^7C_2 \times k$, $k = 1$oe no other terms.
	[Total =] 21	**A1**	Value stated.
		4	

Question	Answer	Marks	Guidance
7(a)(i)	$\left[\frac{104+31}{400}=\right]\frac{135}{400}, \frac{27}{80}$, 0·3375	**B1**	Evaluated, exact value.
		1	
7(a)(ii)	**Method 1**		
	$\mathrm{P}(M)=\frac{180}{400}$, 0·45 $\mathrm{P}(S)=\frac{135}{400}$, 0·3375 $\mathrm{P}(M\cap S)=\frac{31}{400}$, 0·0775 $\frac{180}{400}\times\frac{135}{400}=\frac{243}{1600}, 0.151875\neq\frac{31}{400}$ so NOT independent	**M1**	*Their* P(M) × *their* P(S) seen, accept unsimplified.
		A1	P(M), P(S) and P($M \cap S$) notation seen, numerical comparison and correct conclusion, WWW.
	Method 2		
	$\mathrm{P}(M\cap S)=\frac{31}{400}$ $\mathrm{P}(S)=\frac{135}{400}$ $\mathrm{P}(M)=\frac{180}{400}$ $\mathrm{P}(M\|S)=\frac{\frac{31}{400}}{\frac{135}{400}}=\frac{31}{135}, 0.2296\ldots\neq\frac{180}{400}$ so NOT independent	**M1**	$[\mathrm{P}(M\|S)=]\frac{\textit{their}\,\mathrm{P}(M\cap S)}{\textit{their}\,\mathrm{P}(S)}$ (oe) seen, accept unsimplified.
		A1	P(M), P(S) and P($M \cap S$) notation seen, numerical comparison and correct conclusion, WWW.
		2	

Question	Answer	Marks	Guidance
7(b)(i)	**Method 1** [1 – P(0,1,2)]		
	$= 1 - ({}^{10}C_0\, 0{\cdot}3^0\, 0{\cdot}7^{10} + {}^{10}C_1\, 0{\cdot}3^1\, 0{\cdot}7^9 + {}^{10}C_2\, 0{\cdot}3^2\, 0{\cdot}7^8)$	**M1**	${}^{10}C_x\, p^x\, (1-p)^{10-x}$ for $0 < \times < 10$, $0 < p < 1$, any p.
	$= 1 - (0{\cdot}028248 + 0{\cdot}121061 + 0{\cdot}233474)$	**A1**	Correct expression, accept unsimplified, condone omission of final bracket, condone recovery from poor notation.
	$= 0{\cdot}617$	**A1**	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
	Method 2 [P(3,4,5,6,7,8,9,10) =]		
	${}^{10}C_3\, 0{\cdot}3^3\, 0{\cdot}7^7 + {}^{10}C_4\, 0{\cdot}3^4\, 0{\cdot}7^6 + {}^{10}C_5\, 0{\cdot}3^5\, 0{\cdot}7^5 + {}^{10}C_6\, 0{\cdot}3^6\, 0{\cdot}7^4 + {}^{10}C_7\, 0{\cdot}3^7\, 0{\cdot}7^3 + {}^{10}C_8\, 0{\cdot}3^8\, 0{\cdot}7^2 + {}^{10}C_9\, 0{\cdot}3^9\, 0{\cdot}7^1 + {}^{10}C_{10}\, 0{\cdot}3^{10}\, 0{\cdot}7^0$	**M1**	${}^{10}C_x\, p^x\, (1-p)^{10-x}$ for $0 < \times < 10$, $0 < p < 1$, any p.
		A1	Correct unsimplified expression.
	$= 0{\cdot}617$	**A1**	Accept $0{\cdot}61715 \leqslant p \leqslant 0{\cdot}61722$, WWW.
		3	

Question	Answer	Marks	Guidance
7(b)(ii)	[p = 0·3] Mean = 0·3 × 90 = 27; variance = 0·3 × 90 × 0·7 = 18·9	**B1**	Correct mean and variance, allow unsimplified. Condone $\sigma = 4{\cdot}347$ evaluated.
	$\mathrm{P}(X < 32) = P\left(z < \frac{31.5-27}{\sqrt{18.9}}\right)$	**M1**	Substituting *their* μ and σ (not σ^2, $\sqrt{\sigma}$) into the ±standardising formula with a numerical value for '31·5'.
		M1	Using either 31·5 or 32·5 within a ±standardising formula with numerical values for *their* μ and σ (condone σ^2, $\sqrt{\sigma}$).
	$= \Phi(1.035)$	**M1**	Appropriate area Φ, from standardisation formula P(z<…) in final solution, must be probability.
	= 0·850	**A1**	Allow $0{\cdot}8495 < p \leqslant 0{\cdot}85(0)$, final answer WWW.
		5	