Cambridge International A Level

Cambridge Assessment International Education
Cambridge International Advanced Level

MATHEMATICS **9709/32**

Paper 3 Pure Mathematics **March 2019**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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GENERIC MARKING PRINCIPLE 5: Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).
GENERIC MARKING PRINCIPLE 6: Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question	Answer	Marks	Guidance
1(i)	Use law for the logarithm of a product or quotient	**M1**	
	Use $\log_{10} 100 = 2$ or $10^2 = 100$	**M1**	
	Obtain $x^2 - 4x - 100 = 0$, or equivalent	**A1**	
		3	
1(ii)	Solve a 3-term quadratic equation	**M1**	
	Obtain answer 12.2 only	**A1**	
		2	

Question	Answer	Marks	Guidance
2(i)	Use the iterative formula correctly at least once	**M1**	
	Obtain answer 1.3195	**A1**	
	Show sufficient iterations to 6 d.p. to justify 1.3195 to 4 d.p., or show there is a sign change in (1.31945, 1.31955)	**A1**	
		3	
2(ii)	State $x = \dfrac{2x^6 + 12x}{3x^5 + 8}$, or equivalent	**B1**	
	State answer $\sqrt[5]{4}$, or exact equivalent	**B1**	
		2	

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Question	Answer	Marks	Guidance
3(i)	Use trig formulae and obtain an equation in sin θ and cos θ	**M1**	
	Obtain a correct equation in any form	**A1**	
	Substitute exact trig ratios and obtain an expression for tan θ	**M1**	
	Obtain answer $\tan\theta = \dfrac{2\sqrt{2}-1}{1-\sqrt{6}}$, or equivalent	**A1**	
		4	
3(ii)	State answer, e.g. $\theta = 128.4°$	**B1**	
	State second answer, e.g. $\theta = 308.4°$	**B1 ft**	
		2	

Question	Answer	Marks	Guidance
4	Integrate by parts and reach $ax^{-\frac{1}{2}}\ln x + b\int x^{-\frac{1}{2}}.\dfrac{1}{x}\mathrm{d}x$	**M1***	
	Obtain $-2x^{-\frac{1}{2}}\ln x + 2\int x^{-\frac{1}{2}}.\dfrac{1}{x}\mathrm{d}x$, or equivalent	**A1**	
	Complete the integration, obtaining $-2x^{-\frac{1}{2}}\ln x - 4x^{-\frac{1}{2}}$, or equivalent	**A1**	
	Substitute limits correctly, having integrated twice	**M1(dep*)**	
	Obtain the given answer following full and correct working	**A1**	
		5	

Question	Answer	Marks	Guidance
5	State $\cos y \frac{\mathrm{d}y}{\mathrm{d}x}$ as derivative of $\sin y$	**B1**	
	State correct derivative in terms of x and y, e.g. $\sec^2 x / \cos y$	**B1**	
	State correct derivative in terms of x, e.g. $\frac{\sec^2 x}{\sqrt{1-\tan^2 x}}$	**B1**	
	Use double angle formula	**M1**	
	Obtain the given answer correctly	**A1**	
		5	

Question	Answer	Marks	Guidance
6	Separate variables correctly and attempt integration of at least one side	**B1**	
	Obtain term $-\frac{1}{2y^2}$, or equivalent	**B1**	
	Obtain term $-k\ \mathrm{e}^{-x}$	**B1**	
	Use a pair of limits, e.g. $x = 0$, $y = 1$ to obtain an equation in k and an arbitrary constant c	**M1**	
	Use a second pair of limits, e.g. $x = 1$, $y = \sqrt{\mathrm{e}}$, to obtain a second equation and solve for k or for c	**M1**	
	Obtain $k = \frac{1}{2}$ and $c = 0$	**A1**	
	Obtain final answer $y = \mathrm{e}^{\frac{1}{2}x}$, or equivalent	**A1**	
		7	

Question	Answer	Marks	Guidance
7(a)	Use quadratic formula to solve for z	**M1**	
	Use $i^2 = -1$ throughout	**M1**	
	Obtain correct answer in any form	**A1**	
	Multiply numerator and denominator by 1 – i, or equivalent	**M1**	
	Obtain final answer, e.g. 1 – i	**A1**	
	Obtain second final answer, e.g. $\frac{5}{2}+\frac{1}{2}i$	**A1**	
		6	
7(b)	Show the point representing u in relatively correct position	**B1**	
	Show the horizontal line through $z = i$	**B1**	
	Show correct half-lines from u, one of gradient 1 and the other vertical	**B1ft**	
	Shade the correct region	**B1**	
		4	

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Question	Answer	Marks	Guidance
8(i)	State or imply the form $A+\frac{B}{2+x}+\frac{C}{3-2x}$	**B1**	
	Use a correct method for finding a constant	**M1**	
	Obtain one of $A = 2$, $B = -4$ and $C = 6$	**A1**	
	Obtain a second value	**A1**	
	Obtain the third value	**A1**	
		5	
8(ii)	Use correct method to find the first two terms of the expansion of $(2+x)^{-1}$ or $(3-2x)^{-1}$, or equivalent	**M1**	
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	**A1ft +A1ft**	The ft is on B and C
	Add the value of A to the sum of the expansions	**M1**	
	Obtain final answer $2+\frac{7}{3}x+\frac{7}{18}x^2$	**A1**	
		5	

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Question	Answer	Marks	Guidance
9(i)	State or imply a correct normal vector to either plane, e.g. $2\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, or $\mathbf{i} - 2\mathbf{j} + \mathbf{k}$	**B1**	
	Carry out correct process for evaluating the scalar product of two normal vectors	**M1**	
	Using the correct process for the moduli, divide the scalar product of the two normal vectors by the product of their moduli and evaluate the inverse cosine of the result	**M1**	
	Obtain answer 56.9° or 0.994 radians	**A1**	
		4	
9(ii)	*EITHER*: Carry out a complete strategy for finding a point on the line (call the line l)	**M1**	
	Obtain such a point, e.g. (1, 1, 4)	**A1**	
	EITHER: State a correct equation for a direction vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$ for l, e.g. $2a + 3b - c = 0$	**B1**	
	State a second equation, e.g. $a - 2b + c = 0$, and solve for one ratio, e.g. $a : b$	**M1**	
	Obtain $a : b : c = 1 : -3 : -7$, or equivalent	**A1**	
	State a correct answer, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$	**A1**	
	*OR*1: Attempt to calculate the vector product of the two normal vectors	**M1**	
	Obtain two correct components	**A1**	
	Obtain $\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}$, or equivalent	**A1**	
	State a correct answer, e.g. $\mathbf{r} = \mathbf{i} + \mathbf{j} + 4\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	**A1**	

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Question	Answer	Marks	Guidance
9(ii)	*OR*2: Obtain a second point on l e.g. (0, 4, 11)	**B1**	
	Subtract position vectors and obtain a direction vector for l	**M1**	
	Obtain $\mathbf{i} - 3\mathbf{j} - 7\mathbf{k}$, or equivalent	**A1**	
	State a correct answer, e.g. $\mathbf{r} = 4\mathbf{j} + 11\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	**A1**	
	*OR*3: Express one variable in terms of a second	**M1**	
	Obtain a correct simplified expression, e.g. $y = 4 - 3x$	**A1**	
	Express the third variable in terms of the second	**M1**	
	Obtain a correct simplified expression, e.g. $z = 11 - 7x$	**A1**	
	Form a vector equation for the line	**M1**	
	State a correct answer, e.g. $\mathbf{r} = 4\mathbf{j} + 11\mathbf{k} + \lambda(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	**A1**	
		6	

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Question	Answer	Marks	Guidance
9(ii)	*OR*4: Express one variable in terms of a second	**M1**	
	Obtain a correct simplified expression, e.g. $x = \frac{4}{3} - \frac{y}{3}$	**A1**	
	Express the same variable in terms of the third	**M1**	
	Obtain a correct simplified expression, e.g. $x = \frac{11}{7} - \frac{z}{7}$	**A1**	
	Form a vector equation for the line	**M1**	
	Obtain a correct answer, e.g. $\mathbf{r} = 4\mathbf{j} + 11\mathbf{k} + \mu(\mathbf{i} - 3\mathbf{j} - 7\mathbf{k})$, or equivalent	**A1**	
		6	

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Question	Answer	Marks	Guidance
10(i)	State or imply $\mathrm{d}u = -\sin x\,\mathrm{d}x$	**B1**	
	Using Pythagoras express the integral in terms of u	**M1**	
	Obtain integrand $\pm\sqrt{u}\left(1-u^2\right)$	**A1**	
	Integrate and obtain $-\frac{2}{3}u^{\frac{3}{2}}+\frac{2}{7}u^{\frac{7}{2}}$, or equivalent	**A1**	
	Change limits correctly and substitute correctly in an integral of the form $au^{\frac{3}{2}}+bu^{\frac{7}{2}}$	**M1**	Or substitute original limits correctly in an integral of the form $a(\cos x)^{\frac{3}{2}} + b(\cos x)^{\frac{7}{2}}$
	Obtain answer $\frac{8}{21}$	**A1**	
		6	
10(ii)	Use product rule and chain rule at least once	**M1**	
	Obtain correct derivative in any form	**A1 + A1**	
	Equate derivative to zero and obtain a horizontal equation in integral powers of $\sin x$ and $\cos x$	**M1**	
	Use correct methods to obtain an equation in one trig function	**M1**	
	Obtain $\tan^2 x = 6$, $7\cos^2 x = 1$ or $7\sin^2 x = 6$, or equivalent, and obtain answer 1.183	**A1**	
		6	

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