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MATHEMATICS **9709/12**

Paper 1 Pure Mathematics **March 2017**

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

  - Note: B2 or A2 means that the candidate can earn 2 or 0.
    B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)

CWO Correct Working Only – often written by a 'fortuitous' answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question	Answer	Marks	Guidance
1	$(3k)^2 - 4\times 2\times k$	**M1**	Attempt $b^2 - 4ac$
	$9k^2 - 8k > 0$ soi Allow $9k^2 - 8k \geqslant 0$	**A1**	Must involve correct inequality. Can be implied by correct answers
	0, 8/9 soi	**A1**	
	$k < 0$, $k > 8/9$ (or 0.889)	**A1**	Allow $(-\infty, 0)$, $(8/9, \infty)$
	Total:	**4**	

Question	Answer	Marks	Guidance
2	$5C2\left(\frac{1}{ax}\right)^3\left(2ax^2\right)^2$ soi	**B1**	Seen or implied. Can be part of an expansion.
	$10\times\frac{1}{a^3}\times 4a^2 = 5$ soi	**M1A1**	M1 for identifying relevant term and equating to 5, all correct. Ignore extra x
	$a = 8$ cao	**A1**	
	Total:	**4**	

Question	Answer	Marks	Guidance
3(i)	$V = \frac{1}{12}h^3$ oe	**B1**	
	Total:	**1**	
3(ii)	$\frac{\mathrm{d}V}{\mathrm{d}h} = \frac{1}{4}h^2$ or $\frac{\mathrm{d}h}{\mathrm{d}V} = 4(12v)^{-2/3}$	**M1A1**	Attempt differentiation. Allow incorrect notation for M. For A mark accept *their* letter for volume - but otherwise correct notation. Allow V'
	$\frac{\mathrm{d}h}{\mathrm{d}t} = \frac{\mathrm{d}h}{\mathrm{d}V} \times \frac{\mathrm{d}V}{\mathrm{d}t} \quad = \frac{4}{h^2} \times 20$ soi	**DM1**	Use chain rule correctly with $\frac{\mathrm{d}(V)}{\mathrm{d}t} = 20$. Any equivalent formulation. Accept non-explicit chain rule (or nothing at all)
	$\left(\frac{\mathrm{d}h}{\mathrm{d}t}\right) = \frac{4}{10^2} \times 20 = 0.8$ or equivalent fraction	**A1**	
	Total:	**4**	

Question	Answer	Marks	Guidance
4(i)	$ABC = \pi/2 - \pi/7 = 5\pi/14$. $CBD = \pi - 5\pi/14 = 9\pi/14$	**B1**	**AG** Or other valid exact method.
	Total:	**1**	
4(ii)	$\sin\frac{\pi}{7} = \frac{½BC}{8}$ **or** $\frac{BC}{\sin\frac{2\pi}{7}} = \frac{8}{\sin\frac{5\pi}{14}}$ **or** $BC^2 = 8^2 + 8^2 - 2(8)(8)\cos\frac{2\pi}{7}$	**M1**	
	$BC = 6.94(2)$	**A1**	
	arc CD = *their* $6.94 \times 9\pi/14$	**M1**	Expect 14.02(0)
	arc $CB = 8 \times 2\pi/7$	**M1**	Expect 7.18(1)
	perimeter = 6.94 + 14.02 + 7.18 = 28.1	**A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
5(i)	$\tan x = \cos x \rightarrow \sin x = \cos^2 x$	**M1**	Use tan = sin/cos and multiply by cos
	$\sin x = 1 - \sin^2 x$	**M1**	Use $\cos^2 x = 1 - \sin^2 x$
	$\sin x = 0.6180$. Allow $(-1 + \sqrt{5})/2$	**M1**	Attempt soln of quadratic in $\sin x$. Ignore solution –1.618. Allow x = 0.618
	x-coord of $A = \sin^{-1} 0.618 = 0.666$ cao	**A1**	Must be radians. Accept 0.212π
	Total:	**4**	
5(ii)	***EITHER*** x-coord of B is $\pi - their\, 0.666$	**(M1**	Expect 2.475(3). Must be radians throughout
	y-coord of B is $\tan(their\, 2.475)$ or $\cos(their\, 2.475)$	**M1**	
	$x = 2.48$, $y = -0.786$ or -0.787 cao	**A1)**	Accept $x = 0.788\pi$
	OR y-coord of B is – (cos or tan (*their* 0.666))	**(M1**	
	x-coord of B is $\cos^{-1}$ (*their* y) or $\pi + \tan^{-1}$ (*their* y)	**M1**	
	$x = 2.48$, $y = -0.786$ or -0.787	**A1)**	Accept $x = 0.788\pi$
	Total:	**3**	

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Question	Answer	Marks	Guidance
6(i)	**BA** = **OA** – **OB** = –5**i** – **j** + 2**k**	**B1**	Allow vector reversed. Ignore label **BA** or **AB**
	OA.BA = –10 – 3 + 10 = –3	**M1**	soi by ±3
	$\|\mathbf{OA}\|\times\|\mathbf{BA}\| = \sqrt{2^2+3^2+5^2}\times\sqrt{5^2+1^2+2^2}$	**M1**	Prod. of mods for at least 1 correct vector or reverse.
	$\cos OAB = \frac{+/-3}{\sqrt{38}\times\sqrt{30}}$	**M1**	
	$OAB = 95.1^\circ$ (or 1.66^c)	**A1**	
	Total:	**5**	
6(ii)	$\Delta\, OAB = \frac{1}{2}\sqrt{38}\times\sqrt{30}\sin 95.1$. Allow $\frac{1}{2}\sqrt{38}\times\sqrt{74}\sin 39.4$	**M1**	Allow their moduli product from **(i)**
	= 16.8	**A1**	cao but NOT from sin 84.9 (1.482^c)
	Total:	**2**	

Question	Answer	Marks	Guidance
7(i)	$f'(x) = \left[\frac{3}{2}(4x+1)^{1/2}\right]$ [4]	**B1B1**	Expect $6(4x+1)^{1/2}$ but can be unsimplified.
	$f''(x) = 6\times 1/2\times(4x+1)^{-1/2}\times 4$	**B1**√	Expect $12(4x+1)^{-1/2}$ but can be unsimplified. Ft from *their* $f'(x)$.
	Total:	**3**	
7(ii)	f(2), $f'(2)$, $kf''(2) = 27, 18, 4k$ OR 12	**B1B1**√**B1**√	Ft dependent on attempt at differentiation
	$27/18 = 18/4k$ oe OR $kf''(2) = 12 \Rightarrow k = 3$	**M1A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
8(i)	$\text{gf}(x) = 3\left(2x^2+3\right)+2 = 6x^2+11$	**B1**	**AG**
	$\text{fg}(x) = 2\left(3x+2\right)^2+3$ Allow $18x^2+24x+11$	**B1**	ISW if simplified incorrectly. Not retrospectively from **(ii)**
	Total:	**2**	
8(ii)	$y = 2\left(3x+2\right)^2+3 \quad \Rightarrow 3x+2 = (\pm)\sqrt{(y-3)/2}$ oe	**M1**	Subtract 3;divide by 2;square root. Or *x*/*y* interchanged. Allow $\frac{\sqrt{y-3}}{2}$ for 1st M
	$\Rightarrow x = (\pm)\frac{1}{3}\sqrt{(y-3)/2}-\frac{2}{3}$ oe	**M1**	Subtract 2; divide by 3; Indep. of 1st M1. Or *x*/*y* interchanged.
	$\Rightarrow (\text{fg})^{-1}(x) = \frac{1}{3}\sqrt{(x-3)/2}-\frac{2}{3}$ oe	**A1**	Must be a function of *x*. Allow alt. method $\text{g}^{-1}\text{f}^{-1}(x)$ OR $18\left(x+\frac{2}{3}\right)^2+3 \Rightarrow (\text{fg})^{-1}(x) = \sqrt{\frac{x-3}{18}}-\frac{2}{3}$
	Solve *their* $(\text{fg})^{-1}(x) \geqslant 0$ or attempt range of fg	**M1**	Allow <u>range</u> $\geqslant 3$ for M only. Can be implied by correct answer or $x >$ 11
	Domain is $x \geqslant 11$	**A1**	
	Total:	**5**	

Question	Answer	Marks	Guidance
8(iii)	$6(2x)^2+11=2(3x+2)^2+3$	**M1**	Replace x with $2x$ in gf and equate to *their* fg(x) from (i). Allow $\underline{12}\,x^2+11=$
	$6x^2-24x=0$ oe	**A1**	Collect terms to obtain correct quadratic expression.
	$x=0$, 4	**A1**	Both required
	Total:	**3**	

Question	Answer	Marks	Guidance
9(i)	$\frac{dy}{dx}=2x-2$. At $x=2$, $m=2$	**B1B1**	Numerical m
	Equation of tangent is $y-2=2(x-2)$	**B1**	Expect y = 2x – 2
	Total:	**3**	
9(ii)	Equation of normal $y-2=-½(x-2)$	**M1**	Through (2, 2) with gradient = –1/m . Expect $y=-½x+3$
	$x^2-2x+2=-½x+3 \rightarrow 2x^2-3x-2=0$	**M1**	Equate and simplify to 3-term quadratic
	$x=-½$, $y=3¼$	**A1A1**	Ignore answer of (2, 2)
	Total:	**4**	

Question	Answer	Marks	Guidance
9(iii)	At $x = -½$, grad $= 2(-½) - 2 = -3$	**B1**√	Ft *their* $-½$.
	Equation of tangent is $y - 3¼ = -3(x + ½)$	***M1**	Through *their B* with grad *their* –3 (not m_1 or m_2). Expect $y = -3x + 7/4$
	$2x - 2 = -3x + 7/4$	**DM1**	Equate *their* tangents or attempt to solve simultaneous equations
	$x = 3/4$, $y = -½$	**A1**	Both required.
	Total:	**4**	

Question	Answer	Marks	Guidance
10(i)	$2x - 2/x^3 = 0$	**M1**	Set = 0.
	$x^4 = 1 \Rightarrow x = 1$ at A cao	**A1**	Allow 'spotted' $x = 1$
	Total:	**2**	
10(ii)	$f(x) = x^2 + 1/x^2 (+c)$ cao	**B1**	
	$\frac{189}{16} = 16 + 1/16 + c$	**M1**	Sub $(4, \frac{189}{16})$. c must be present. Dep. on integration
	$c = -17/4$	**A1**	
	Total:	**3**	

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Question	Answer	Marks	Guidance
10(iii)	$x^2+1/x^2-17/4=0 \Rightarrow 4x^4-17x^2+4\ (=0)$	**M1**	Multiply by $4x^2$ (or similar) to transform into 3-term quartic.
	$(4x^2-1)(x^2-4)\ (=0)$	**M1**	Treat as quadratic in x^2 and attempt solution or factorisation.
	$x=½,\ 2$	**A1A1**	Not necessary to distinguish. Ignore negative values. No working scores 0/4
	Total:	**4**	
10(iv)	$\int(x^2+x^{-2}-17/4)\mathrm{d}x\ =\ \frac{x^3}{3}-\frac{1}{x}-\frac{17x}{4}$	**B2,1,0**	Mark final integral
	$(8/3-1/2-17/2)-(1/24-2-17/8)$	**M1**	Apply *their* limits from **(iii)** (Seen). Dep. on integration of at least 1 term of y
	Area = $9/4$	**A1**	Mark final answer. $\int y^2$ scores 0/4
	Total:	**4**	