* 0000800000001 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

* 0 7 9 2 9 3 3 9 4 1 *

FURTHER MATHEMATICS **9231/43**

Paper 4 Further Probability & Statistics **October/November 2024**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 Ellie is investigating the heights of two types of beech tree, A and B, in a certain region. She has chosen a random sample of 60 beech trees of type A in the region, recorded their heights, x m, and calculated unbiased estimates for the population mean and population variance as 35.6 m and 4.95 m^2 respectively.

Ellie also chooses a random sample of 50 beech trees of type B in the region and records their heights, y m. Her results are summarised as follows.

$$\Sigma y = 1654 \qquad \Sigma y^2 = 54\,850$$

Find a 95% confidence interval for the difference between the population mean heights of type A and type B beech trees in the region. [6]

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2 A school with a large number of students is updating its logo. Each student has designed a new logo and two teachers have each awarded a mark out of 50 for each logo. The marks awarded to a random sample of 12 students are shown in the following table.

Student	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*	*K*	*L*
Teacher 1	36	38	40	36	22	34	45	44	48	35	28	30
Teacher 2	38	42	32	41	32	41	42	50	36	44	42	41

One of the students claims that Teacher 2 is awarding higher marks than Teacher 1.

(a) Carry out a Wilcoxon matched-pairs signed-rank test, at the 5% significance level, to test whether the data supports the claim. [7]

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It was later discovered that Teacher 1 had entered her mark for student *C* incorrectly. Her intended mark was 24 not 40. This was corrected.

(b) Determine whether this correction affects the conclusion of the test carried out in part **(a)**. [2]

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3 A statistician believes that the number of telephone calls received by an advice centre in a 10-minute interval can be modelled by the Poisson distribution Po(1.9). The number of calls received in a randomly chosen 10-minute interval was recorded on each of 100 days. The results are summarised in the table, together with some of the expected frequencies corresponding to the distribution Po(1.9).

Number of calls	0	1	2	3	4	5	6 or more
Observed frequency	10	18	35	21	11	4	1
Expected frequency	14.957	28.418	26.997				1.322

(a) Complete the table. [2]

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(b) Carry out a goodness of fit test, at the 10% significance level, to determine whether the statistician's belief is reasonable. [6]

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4 The continuous random variable X has probability density function f given by

$$\mathrm{f}(x) = \begin{cases} kx^3 & 0 \leqslant x < 1, \\ k(5-x) & 1 \leqslant x \leqslant 5, \\ 0 & \text{otherwise}, \end{cases}$$

where k is a constant.

(a) Sketch the graph of f. [1]

(b) Show that $k = \frac{4}{33}$. [2]

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(c) Find the cumulative distribution function of X. [3]

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(d) Find the median value of X. [4]

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5 Nikita has three coins. One coin is fair, one coin is biased so that the probability of obtaining a head is $\frac{1}{3}$ and the third coin is biased so that the probability of obtaining a head is $\frac{1}{5}$. The random variable X is the number of heads that Nikita obtains when he throws all three coins at the same time.

(a) Find the probability generating function of X. [3]

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Rajesh has two fair six-sided dice with faces labelled 1, 2, 3, 4, 5, 6. The random variable Y is the number of 4s that Rajesh obtains when he throws the two dice.

The random variable Z is the sum of the number of heads obtained by Nikita and the number of 4s obtained by Rajesh.

(b) Find the probability generating function of Z, expressing your answer as a polynomial. [4]

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(c) Use your answer to part **(b)** to find E(Z). [2]

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6 Ansal is investigating the wingspans of Monarch butterflies in two different regions, X and Y. He takes a random sample of 8 Monarch butterflies from region X and records their wingspans, x cm. His results are as follows.

8.2 7.0 7.3 8.8 7.8 8.5 9.2 7.4

Ansal also takes a random sample of 9 Monarch butterflies from region Y and records their wingspans, y cm. His results are summarised as follows.

$$\Sigma y = 71.10 \qquad \Sigma y^2 = 567.13$$

Ansal suspects that the mean wingspan of Monarch butterflies from region X is greater than the mean wingspan of Monarch butterflies from region Y. It is known that the wingspans of Monarch butterflies in regions X and Y are normally distributed with equal population variances.

Test, at the 10% significance level, whether Ansal's suspicion is supported by the data. [8]

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