* 0000800000001 *

Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

5412066136

FURTHER MATHEMATICS **9231/23**

Paper 2 Further Pure Mathematics 2 **October/November 2024**

2 hours

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Any blank pages are indicated.

DC (LO) 344630
© UCLES 2024

BLANK PAGE

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

1 Find the set of values of k for which the system of equations

$$\begin{aligned} x + 5y + 6z &= 1, \\ kx + 2y + 2z &= 2, \\ -3x + 4y + 8z &= 3, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [4]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

2 It is given that

$$x = 1 + \frac{1}{t} \quad \text{and} \quad y = \cos^{-1} t \quad \text{for } 0 < t < 1.$$

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{t^2}{\sqrt{1-t^2}}$. [2]

DO NOT WRITE IN THIS MARGIN

(b) Show that $\frac{d^2y}{dx^2} = -t^a(1-t^2)^b(2-t^2)$, where a and b are constants to be determined. [4]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

3 A curve has equation $y = e^x$ for $\ln\frac{4}{3} \leqslant x \leqslant \ln\frac{12}{5}$. The area of the surface generated when the curve is rotated through 2π radians about the x-axis is denoted by A.

(a) Use the substitution $u = e^x$ to show that

$$A = 2\pi \int_{\frac{4}{3}}^{\frac{12}{5}} \sqrt{1+u^2}\,\mathrm{d}u. \qquad [2]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Use the substitution $u = \sinh v$ to show that

$$A = \pi\left(\frac{904}{225} + \ln\frac{5}{3}\right). \qquad [6]$$

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

4 The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} -11 & 1 & 8 \\ 0 & -2 & 0 \\ -16 & 1 & 13 \end{pmatrix}.$$

(a) Show that $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is an eigenvector of **A** and state the corresponding eigenvalue. [2]

.....

.....

.....

.....

.....

.....

.....

.....

(b) Show that the characteristic equation of **A** is $\lambda^3 - 19\lambda - 30 = 0$ and hence find the other eigenvalues of **A**. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN

(c) Use the characteristic equation of **A** to find $\mathbf{A}^{-1}$. [4]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

5 Find the particular solution of the differential equation

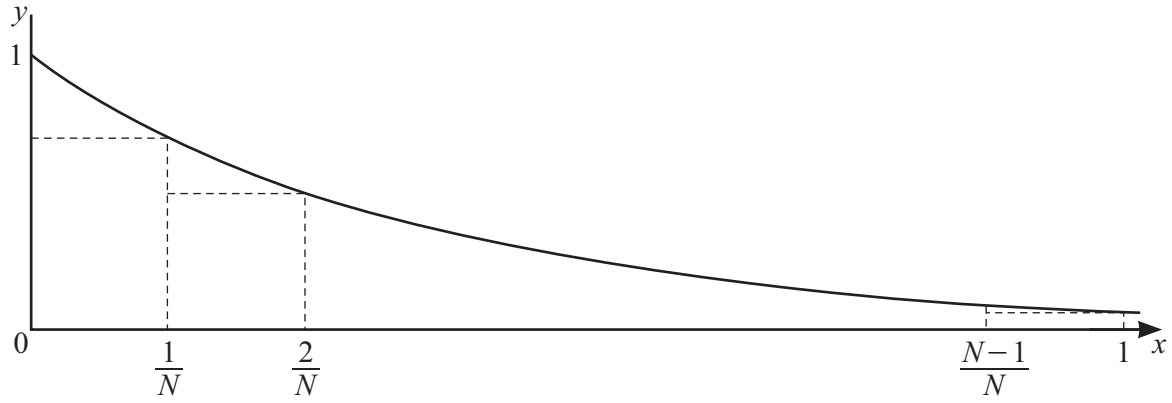
$$6\frac{\mathrm{d}^2x}{\mathrm{d}t^2} - 5\frac{\mathrm{d}x}{\mathrm{d}t} + x = t^2 + t + 1,$$

given that, when $t = 0$, $x = 12$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = -6$. [10]

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

6



The diagram shows the curve with equation $y = \left(\frac{1}{2}\right)^x$ for $0 \leqslant x \leqslant 1$, together with a set of N rectangles each of width $\frac{1}{N}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \left(\frac{1}{2}\right)^x \mathrm{d}x > L_N$, where

$$L_N = \frac{1}{2N\left(2^{\frac{1}{N}} - 1\right)}.$$ [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

(b) Use a similar method to find, in terms of N, an upper bound U_N for $\int_0^1 \left(\frac{1}{2}\right)^x \mathrm{d}x$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(c) Find the least value of N such that $U_N - L_N \leqslant 10^{-3}$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

(d) Given that $\int_0^1 \left(\frac{1}{2}\right)^x \mathrm{d}x = \frac{1}{2\ln 2}$, use the value of N found in part **(c)** to find upper and lower bounds for $\ln 2$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

7 **(a)** Show that an appropriate integrating factor for

$$\sqrt{x^2+16}\,\frac{\mathrm{d}y}{\mathrm{d}x} + y = x\sqrt{x^2+16}$$

is $\frac{1}{4}x + \frac{1}{4}\sqrt{x^2+16}$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN

(b) Hence find the solution of the differential equation

$$\sqrt{x^2+16}\,\frac{\mathrm{d}y}{\mathrm{d}x}+y=x\sqrt{x^2+16}$$

for which $y=6$ when $x=3$. [6]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

BLANK PAGE

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

8 **(a)** By considering the binomial expansion of $\left(z+\frac{1}{z}\right)^7$, where $z=\cos\theta+\mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\cos^7\theta = a\cos 7\theta + b\cos 5\theta + c\cos 3\theta + d\cos\theta,$$

where a, b, c and d are constants to be determined. [5]

Let $I_n = \int_0^{\frac{1}{4}\pi} \cos^n \theta \, \mathrm{d}\theta$.

(b) Show that

$$nI_n = 2^{-\frac{1}{2}n} + (n-1)I_{n-2}.$$ [4]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

(c) Using the results given in parts **(a)** and **(b)**, find the exact value of I_9. [5]

DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN
DO NOT WRITE IN THIS MARGIN

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

DO NOT WRITE IN THIS MARGIN

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.