Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME									
CENTRE NUMBER						CANDIDATE NUMBER				

0815246258

FURTHER MATHEMATICS **9231/42**

Paper 4 Further Probability & Statistics **October/November 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A factory produces small bottles of natural spring water. Two different machines, X and Y, are used to fill empty bottles with the water. A quality control engineer checks the volumes of water in the bottles filled by each of the machines. He chooses a random sample of 60 bottles filled by machine X and a random sample of 75 bottles filled by machine Y. The volumes of water, x and y respectively, in millilitres, are summarised as follows.

$$\Sigma x = 6345 \quad \Sigma(x-\overline{x})^2 = 243.8 \quad \Sigma y = 7614 \quad \Sigma(y-\overline{y})^2 = 384.9$$

$\overline{x}$ and $\overline{y}$ are the sample means of the volume of water in the bottles filled by machines X and Y respectively.

Find a 95% confidence interval for the difference between the mean volume of water in bottles filled by machine X and the mean volume of water in bottles filled by machine Y. [6]

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2 The number of breakdowns on a particular section of road is recorded each day over a period of 90 days. It is suggested that the number of breakdowns follows a Poisson distribution with mean 3.5. The data is summarised in the table, together with some of the expected frequencies resulting from the suggested Poisson distribution.

Number of breakdowns per day	0	1	2	3	4	5	6	7	8 or more
Observed frequency	0	5	13	17	21	16	9	5	4
Expected frequency	2.718	9.512	16.646		16.993	11.895		3.469	2.407

(a) Complete the table. [2]

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(b) Carry out a goodness of fit test, at the 10% significance level, to determine whether or not Po(3.5) is a good fit to the data. [6]

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3 Toby has a bag which contains 6 red marbles and 3 green marbles. He randomly chooses 3 marbles from the bag, without replacement. The random variable X is the number of red marbles that Toby obtains.

(a) Find the probability generating function of X. [3]

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Ling also has a bag which contains 6 red marbles and 3 green marbles. He randomly chooses 2 marbles from his bag, without replacement. The random variable Y is the number of red marbles that Ling obtains. It is given that the probability generating function of Y is $\frac{1}{12}(1+6t+5t^2)$.

The random variable Z is the total number of red marbles that Toby and Ling obtain.

(b) Find the probability generating function of Z, expressing your answer as a polynomial in t. [3]

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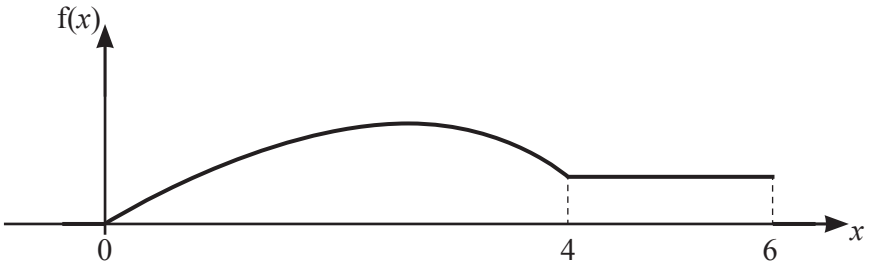
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(c) Use the probability generating function of Z to find $\text{Var}(Z)$. [4]

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The diagram shows the continuous random variable X with probability density function f given by

$$\mathrm{f}(x) = \begin{cases} \frac{1}{128}(4ax - bx^3) & 0 \leqslant x \leqslant 4, \\ c & 4 \leqslant x \leqslant 6, \\ 0 & \text{otherwise}, \end{cases}$$

where a, b and c are constants.

The upper quartile of X is equal to 4.

(a) Show that $c = \frac{1}{8}$ and find the values of a and b. [4]

(b) Find the exact value of the median of X. [3]

(c) Find $\mathrm{E}(\sqrt{X})$, giving your answer correct to 2 decimal places. [3]

5 A company is deciding which of two machines, X and Y, can make a certain type of electrical component more quickly. The times taken, in minutes, to make one component of this type are recorded for a random sample of 8 components made by machine X and a random sample of 9 components made by machine Y. These times are as follows.

Machine X	4.0	4.6	4.7	4.8	5.0	5.2	5.6	5.8	
Machine Y	4.5	4.9	5.1	5.3	5.4	5.7	5.9	6.3	6.4

The manager claims that on average the time taken by machine X to make one component is less than that taken by machine Y.

(a) Carry out a Wilcoxon rank-sum test at the 5% significance level to test whether the manager's claim is supported by the data. [6]

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(b) Assuming that the times taken to produce the components by the two machines are normally distributed with equal variances, carry out a *t*-test at the 5% significance level to test whether the manager's claim is supported by the data. [9]

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Question 5(c) is printed on the next page.

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(c) In general, would you expect the conclusions from the tests in parts **(a)** and **(b)** to be the same? Give a reason for your answer. [1]

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Additional page

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