Cambridge Assessment International Education

# Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

CANDIDATE NUMBER | | | | |

*6541057198*

**FURTHER MATHEMATICS** **9231/33**

Paper 3 Further Mechanics **October/November 2023**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ($g$) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

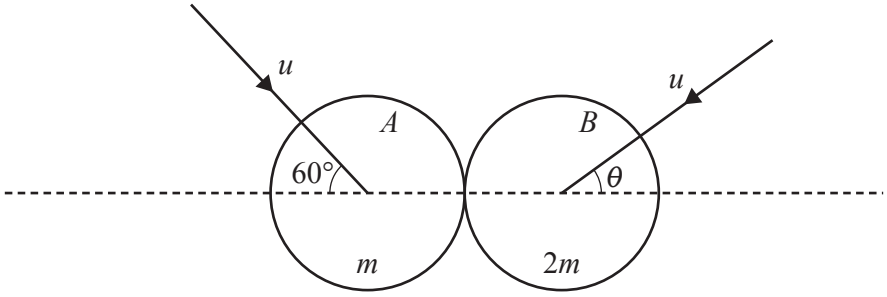
**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.


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**1**




Two uniform smooth spheres $A$ and $B$ of equal radii have masses $m$ and $2m$ respectively. The two spheres are moving with equal speeds $u$ on a smooth horizontal surface when they collide. Immediately before the collision, $A$'s direction of motion makes an angle of 60° with the line of centres, and $B$'s direction of motion makes an angle $\theta$ with the line of centres (see diagram). The coefficient of restitution between the spheres is $e$.

After the collision, the component of the velocity of $A$ along the line of centres is $v$ and $B$ moves perpendicular to the line of centres. Sphere $A$ now has twice as much kinetic energy as sphere $B$.

**(a)** Show that $v = \frac{1}{2}u(4\cos\theta - 1)$. [1]

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**(b)** Find the value of $\cos\theta$. [4]

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**(c)** Find the value of $e$. [2]

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**2** A ball of mass 2 kg is projected vertically downwards with speed $5\,\text{m}\,\text{s}^{-1}$ through a liquid. At time $t$ s after projection, the velocity of the ball is $v\,\text{m}\,\text{s}^{-1}$ and its displacement from its starting point is $x$ m. The forces acting on the ball are its weight and a resistive force of magnitude $0.2v^2$ N.

**(a)** Find an expression for $v$ in terms of $t$. [6]

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**(b)** Deduce what happens to $v$ for large values of $t$. [1]

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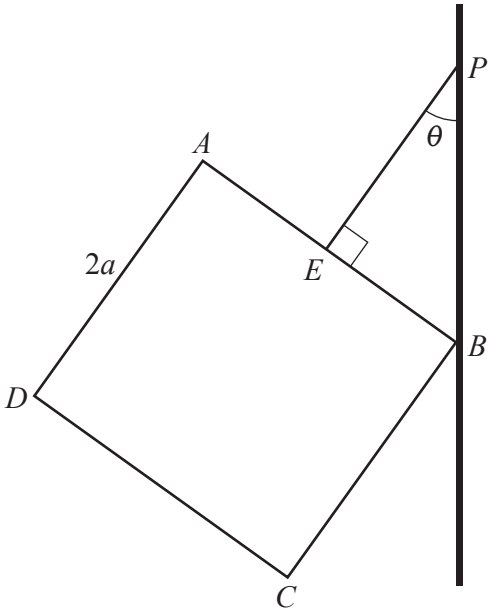
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**3**




A uniform square lamina of side $2a$ and weight $W$ is suspended from a light inextensible string attached to the midpoint $E$ of the side $AB$. The other end of the string is attached to a fixed point $P$ on a rough vertical wall. The vertex $B$ of the lamina is in contact with the wall. The string $EP$ is perpendicular to the side $AB$ and makes an angle $\theta$ with the wall (see diagram). The string and the lamina are in a vertical plane perpendicular to the wall. The coefficient of friction between the wall and the lamina is $\frac{1}{2}$.

Given that the vertex $B$ is about to slip up the wall, find the value of $\tan\theta$. [8]

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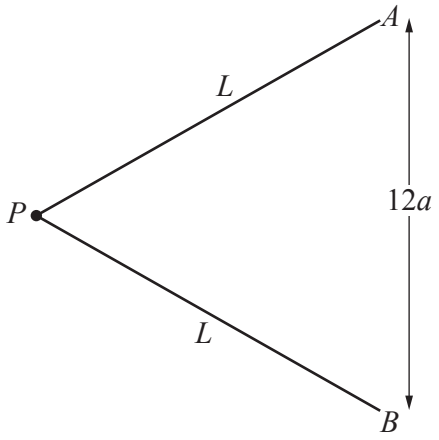
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**4**




A light elastic string has natural length $8a$ and modulus of elasticity $5mg$. A particle $P$ of mass $m$ is attached to the midpoint of the string. The ends of the string are attached to points $A$ and $B$ which are a distance $12a$ apart on a smooth horizontal table. The particle $P$ is held on the table so that $AP = BP = L$ (see diagram). The particle $P$ is released from rest. When $P$ is at the midpoint of $AB$ it has speed $\sqrt{80ag}$.

**(a)** Find $L$ in terms of $a$. [5]

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**(b)** Find the initial acceleration of $P$ in terms of $g$. [3]

**5** A particle $P$ is projected with speed $u\,\mathrm{m\,s^{-1}}$ at an angle $\theta$ above the horizontal from a point $O$ on a horizontal plane and moves freely under gravity. During its flight $P$ passes through the point which is a horizontal distance $3a$ from $O$ and a vertical distance $\frac{3}{8}a$ above the horizontal plane. It is given that $\tan\theta = \frac{1}{3}$.

**(a)** Show that $u^2 = 8ag$. [2]

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A particle $Q$ is projected with speed $V\,\mathrm{m\,s^{-1}}$ at an angle $\alpha$ above the horizontal from $O$ at the instant when $P$ is at its highest point. Particles $P$ and $Q$ both land at the same point on the horizontal plane at the same time.

**(b)** Find $V$ in terms of $a$ and $g$. [7]

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**6** A particle $P$ of mass $m$ is attached to one end of a light inextensible rod of length $3a$. An identical particle $Q$ is attached to the other end of the rod. The rod is smoothly pivoted at a point $O$ on the rod, where $OQ = x$. The system, of rod and particles, rotates about $O$ in a vertical plane.

At an instant when the rod is vertical, with $P$ above $Q$, the particle $P$ is moving horizontally with speed $u$. When the rod has turned through an angle of 60° from the vertical, the speed of $P$ is $2\sqrt{ag}$, and the tensions in the two parts of the rod, $OP$ and $OQ$, have equal magnitudes.

**(a)** Show that the speed of $Q$ when the rod has turned through an angle of 60° from the vertical is $\frac{2x}{3a-x}\sqrt{ag}$. [2]

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**(b)** Find $x$ in terms of $a$. [5]

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**(c)** Find $u$ in terms of $a$ and $g$. [4]

## Additional page

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