Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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1394054656

FURTHER MATHEMATICS **9231/32**

Paper 3 Further Mechanics **October/November 2023**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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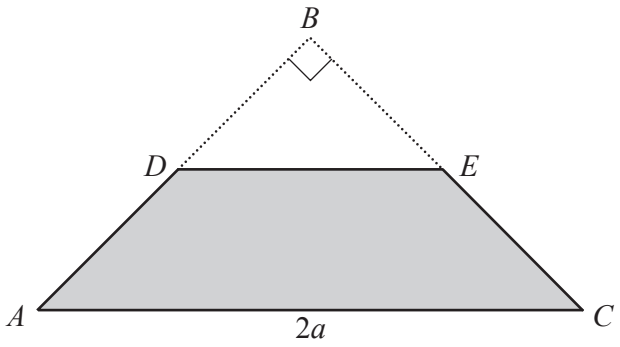
1 One end of a light inextensible string of length a is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The string is taut and makes an angle θ with the downward vertical through O, where $\cos\theta = \frac{2}{3}$. The particle moves in a horizontal circle with speed v.

Find v in terms of a and g. [4]

2 A particle P of mass 0.5 kg moves in a straight line. At time t s the velocity of P is $v\,\text{m}\,\text{s}^{-1}$ and its displacement from a fixed point O on the line is x m. The only forces acting on P are a force of magnitude $\frac{150}{(x+1)^2}$ N in the direction of increasing displacement and a resistive force of magnitude $\frac{450}{(x+1)^3}$ N. When $t = 0$, $x = 0$ and $v = 20$.

Find v in terms of x, giving your answer in the form $v = \frac{Ax+B}{(x+1)}$, where A and B are constants to be determined. [6]

3



A uniform lamina is in the form of an isosceles triangle ABC in which $AC = 2a$ and angle $ABC = 90°$. The point D on AB is such that the ratio $DB : AB = 1 : k$. The point E on CB is such that DE is parallel to AC. The triangle DBE is removed from the lamina (see diagram).

(a) Find, in terms of k, the distance of the centre of mass of the remaining lamina $ADEC$ from the midpoint of AC. [4]

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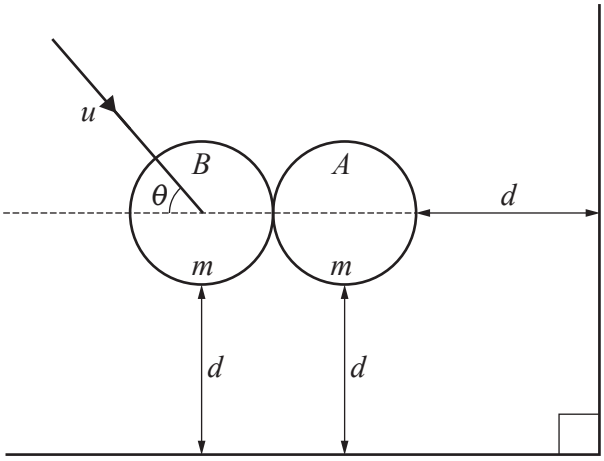
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When the lamina $ADEC$ is freely suspended from the vertex A, the edge AC makes an angle θ with the downward vertical, where $\tan\theta = \frac{5}{18}$.

(b) Find the value of k. [3]

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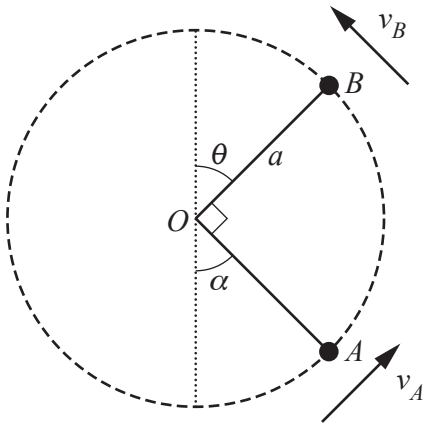


Two smooth vertical walls meet at right angles. The smooth sphere A, with mass m, is at rest on a smooth horizontal surface and is at a distance d from each wall. An identical smooth sphere B is moving on the horizontal surface with speed u at an angle θ with the line of centres when the spheres collide (see diagram). After the collision, the spheres take the same time to reach a wall. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(a) Find the value of $\tan\theta$. [4]

(b) Find the percentage loss in the total kinetic energy of the spheres as a result of this collision. [3]

5



A bead of mass m moves on a smooth circular wire, with centre O and radius a, in a vertical plane. The bead has speed v_A when it is at the point A where OA makes an angle α with the downward vertical through O, and $\cos\alpha = \frac{3}{5}$. Subsequently the bead has speed v_B at the point B, where OB makes an angle θ with the upward vertical through O. Angle AOB is a right angle (see diagram). The reaction of the wire on the bead at B is in the direction OB and has magnitude equal to $\frac{1}{6}$ of the magnitude of the reaction when the bead is at A.

(a) Find, in terms of m and g, the magnitude of the reaction at B. [6]

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(b) Given that $v_A = \sqrt{kag}$, find the value of k. [2]

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6 A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$ [3]

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During its flight, P must clear an obstacle of height h m that is at a horizontal distance of 32 m from the point of projection. When $u = 40\sqrt{2}\,\text{m s}^{-1}$, P just clears the obstacle. When $u = 40\,\text{m s}^{-1}$, P only achieves 80% of the height required to clear the obstacle.

(b) Find the two possible values of h. [6]

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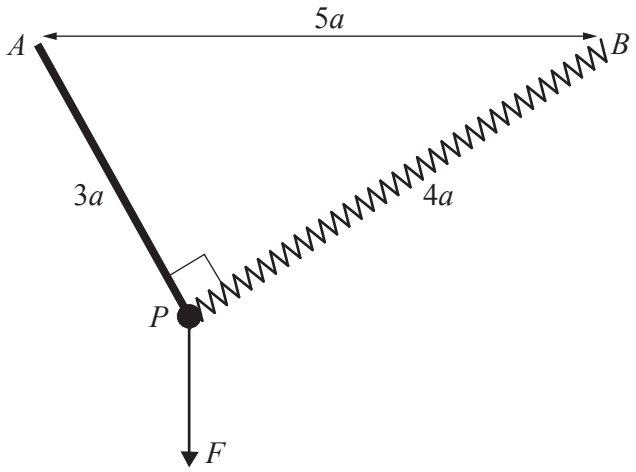
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A particle P of mass m is attached to one end of a light rod of length $3a$. The other end of the rod is able to pivot smoothly about the fixed point A. The particle is also attached to one end of a light spring of natural length a and modulus of elasticity kmg. The other end of the spring is attached to a fixed point B. The points A and B are in a horizontal line, a distance $5a$ apart, and these two points and the rod are in a vertical plane.

Initially, P is held in equilibrium by a vertical force F with the stretched length of the spring equal to $4a$ (see diagram). The particle is released from rest in this position and has a speed of $\frac{6}{5}\sqrt{2ag}$ when the rod becomes horizontal.

(a) Find the value of k. [5]

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(b) Find F in terms of m and g. [2]

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(c) Find, in terms of m and g, the tension in the rod immediately before it is released. [2]

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Additional page

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