Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER | | | | | |

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8489354973

FURTHER MATHEMATICS **9231/33**

Paper 3 Further Mechanics **October/November 2020**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

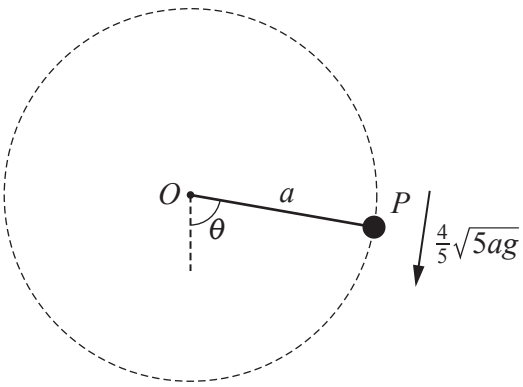
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1 A particle P of mass m is placed on a fixed smooth plane which is inclined at an angle θ to the horizontal. A light spring, of natural length a and modulus of elasticity $3mg$, has one end attached to P and the other end attached to a fixed point O at the top of the plane. The spring lies along a line of greatest slope of the plane. The system is released from rest with the spring at its natural length.

Find, in terms of a and θ, an expression for the greatest extension of the spring in the subsequent motion. [3]

2



A particle P is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle P is held with the string taut and making an angle θ with the downward vertical. The particle P is then projected with speed $\frac{4}{5}\sqrt{5ag}$ perpendicular to the string and just completes a vertical circle (see diagram).

Find the value of $\cos\theta$. [5]

3 One end of a light elastic string, of natural length a and modulus of elasticity $4mg$, is attached to a fixed point O. The other end of the string is attached to a particle of mass m. The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle θ to the downward vertical through O. The length of the string during this motion is $(k+1)a$.

(a) Find the value of k. [4]

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(b) Find the value of $\cos\theta$. [2]

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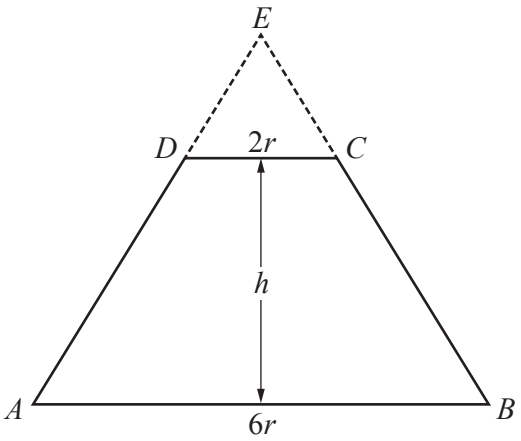
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The diagram shows the cross-section $ABCD$ of a uniform solid object which is formed by removing a cone with cross-section DCE from the top of a larger cone with cross-section ABE. The perpendicular distance between AB and DC is h, the diameter AB is $6r$ and the diameter DC is $2r$.

(a) Find an expression, in terms of h, for the distance of the centre of mass of the solid object from AB. [4]

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The object is freely suspended from the point B and hangs in equilibrium. The angle between AB and the downward vertical through B is θ.

(b) Given that $h = \frac{13}{4}r$, find the value of $\tan\theta$. [2]

5 A particle P is projected with speed u at an angle α above the horizontal from a point O on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of P from O at a subsequent time t are denoted by x and y respectively.

(a) Derive the equation of the trajectory of P in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$ [3]

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The point Q is the highest point on the trajectory of P in the case where $\alpha = 45°$.

(b) Show that the x-coordinate of Q is $\frac{u^2}{2g}$. [3]

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(c) Find the other value of α for which P would pass through the point Q. [4]

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6 Two smooth spheres A and B have equal radii and masses m and $2m$ respectively. Sphere B is at rest on a smooth horizontal floor. Sphere A is moving on the floor with velocity u and collides directly with B. The coefficient of restitution between the spheres is e.

(a) Find, in terms of u and e, the velocities of A and B after the collision. [3]

Subsequently, B collides with a fixed vertical wall which makes an angle θ with the direction of motion of B, where $\tan\theta = \frac{3}{4}$.

The coefficient of restitution between B and the wall is $\frac{2}{3}$. Immediately after B collides with the wall, the kinetic energy of A is $\frac{5}{32}$ of the kinetic energy of B.

(b) Find the possible values of e. [7]

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[Turn over

7 A particle P moving in a straight line has displacement x m from a fixed point O on the line at time t s. The acceleration of P, in $\mathrm{m\,s^{-2}}$, is given by $\frac{200}{x^2} - \frac{100}{x^3}$ for $x > 0$. When $t = 0$, $x = 1$ and P has velocity $10\,\mathrm{m\,s^{-1}}$ directed towards O.

(a) Show that the velocity $v\,\mathrm{m\,s^{-1}}$ of P is given by $v = \frac{10(1-2x)}{x}$. [5]

(b) Show that x and t are related by the equation $e^{-40t} = (2x-1)e^{2x-2}$ and deduce what happens to x as t becomes large. [5]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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