Cambridge Assessment International Education

# Cambridge International AS & A Level

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*4618189778*

**FURTHER MATHEMATICS** **9231/31**

Paper 3 Further Mechanics **October/November 2020**

**1 hour 30 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

**INSTRUCTIONS**

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity ($g$) is needed, use $10\,\text{m}\,\text{s}^{-2}$.

**INFORMATION**

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

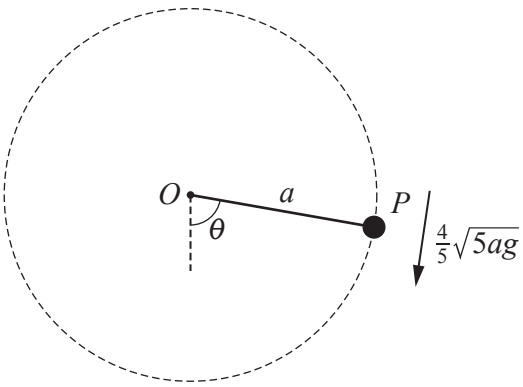

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**1** A particle $P$ of mass $m$ is placed on a fixed smooth plane which is inclined at an angle $\theta$ to the horizontal. A light spring, of natural length $a$ and modulus of elasticity $3mg$, has one end attached to $P$ and the other end attached to a fixed point $O$ at the top of the plane. The spring lies along a line of greatest slope of the plane. The system is released from rest with the spring at its natural length.

Find, in terms of $a$ and $\theta$, an expression for the greatest extension of the spring in the subsequent motion. [3]

**2**




A particle $P$ is attached to one end of a light inextensible string of length $a$. The other end of the string is attached to a fixed point $O$. The particle $P$ is held with the string taut and making an angle $\theta$ with the downward vertical. The particle $P$ is then projected with speed $\frac{4}{5}\sqrt{5ag}$ perpendicular to the string and just completes a vertical circle (see diagram).

Find the value of $\cos\theta$. [5]

**3** One end of a light elastic string, of natural length $a$ and modulus of elasticity $4mg$, is attached to a fixed point $O$. The other end of the string is attached to a particle of mass $m$. The particle moves in a horizontal circle with a constant angular speed $\sqrt{\frac{g}{a}}$ with the string inclined at an angle $\theta$ to the downward vertical through $O$. The length of the string during this motion is $(k+1)a$.

**(a)** Find the value of $k$. [4]

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**(b)** Find the value of $\cos\theta$. [2]

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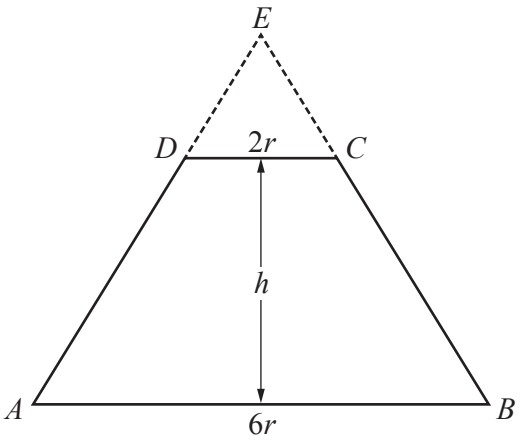
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**4**




The diagram shows the cross-section $ABCD$ of a uniform solid object which is formed by removing a cone with cross-section $DCE$ from the top of a larger cone with cross-section $ABE$. The perpendicular distance between $AB$ and $DC$ is $h$, the diameter $AB$ is $6r$ and the diameter $DC$ is $2r$.

**(a)** Find an expression, in terms of $h$, for the distance of the centre of mass of the solid object from $AB$. [4]

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The object is freely suspended from the point $B$ and hangs in equilibrium. The angle between $AB$ and the downward vertical through $B$ is $\theta$.

**(b)** Given that $h = \frac{13}{4}r$, find the value of $\tan\theta$. [2]

**5** A particle $P$ is projected with speed $u$ at an angle $\alpha$ above the horizontal from a point $O$ on a horizontal plane and moves freely under gravity. The horizontal and vertical displacements of $P$ from $O$ at a subsequent time $t$ are denoted by $x$ and $y$ respectively.

**(a)** Derive the equation of the trajectory of $P$ in the form

$$y = x\tan\alpha - \frac{gx^2}{2u^2}\sec^2\alpha.$$ [3]

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The point $Q$ is the highest point on the trajectory of $P$ in the case where $\alpha = 45°$.

**(b)** Show that the $x$-coordinate of $Q$ is $\frac{u^2}{2g}$. [3]

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**(c)** Find the other value of $\alpha$ for which $P$ would pass through the point $Q$. [4]

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**6** Two smooth spheres $A$ and $B$ have equal radii and masses $m$ and $2m$ respectively. Sphere $B$ is at rest on a smooth horizontal floor. Sphere $A$ is moving on the floor with velocity $u$ and collides directly with $B$. The coefficient of restitution between the spheres is $e$.

**(a)** Find, in terms of $u$ and $e$, the velocities of $A$ and $B$ after the collision. [3]

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Subsequently, $B$ collides with a fixed vertical wall which makes an angle $\theta$ with the direction of motion of $B$, where $\tan\theta = \frac{3}{4}$.

The coefficient of restitution between $B$ and the wall is $\frac{2}{3}$. Immediately after $B$ collides with the wall, the kinetic energy of $A$ is $\frac{5}{32}$ of the kinetic energy of $B$.

**(b)** Find the possible values of $e$. [7]

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**7** A particle $P$ moving in a straight line has displacement $x$ m from a fixed point $O$ on the line at time $t$ s. The acceleration of $P$, in $\text{m s}^{-2}$, is given by $\frac{200}{x^2} - \frac{100}{x^3}$ for $x > 0$. When $t = 0$, $x = 1$ and $P$ has velocity $10\,\text{m s}^{-1}$ directed towards $O$.

**(a)** Show that the velocity $v\,\text{m s}^{-1}$ of $P$ is given by $v = \frac{10(1-2x)}{x}$. [5]

**(b)** Show that $x$ and $t$ are related by the equation $e^{-40t} = (2x-1)e^{2x-2}$ and deduce what happens to $x$ as $t$ becomes large. [5]

## Additional Page

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